

# DESIGN AND USE OF EXTERNALLY BONDED FIBRE REINFORCED CARBON POLYMER REINFORCEMENT (CFRP EBR) FOR REINFORCED CONCRETE STRUCTURES



## **Research Activity**

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## INDEX

	Page
1. Introduction.....	4
2. Reasons / Disadvantages.....	5
2.1 Reasons.....	5
2.2 Disadvantages.....	5
3. Applications externally bonded CFRP reinforcement (EBR).....	6
4. Materials.....	7
4.1 Adhesive.....	7
4.2 Matrices.....	8
4.3 Fibres.....	8
4.4 CFRP materials.....	8
5. Technique.....	9
6. General requirements.....	11
7. Practical execution.....	12
7.1 Repair.....	12
7.2 Preparation surface.....	12
7.3 CFRP EBR application.....	14
7.4 Finishing.....	16
7.5 Practical example.....	16
8. Quality control.....	20
8.1 Strength material.....	20
8.2 Qualification workers.....	24
8.3 Quality control practical execution.....	24
8.4 Bond quality after practical execution.....	26
8.5 In-service inspection and maintenance.....	27
9. Examples	
9.1 Beam simply support – Flexural strengthening.....	28
9.2 Beam simply support – Anchored (Max. CFRP force and length).....	37
9.3 Beam simply support – Shear strengthening.....	38
9.4 Circular column – Confined capacity comparison.....	42
9.5 Slab one direction – Flexural strengthening.....	45
10. Bibliography.....	55

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# 1. INTRODUCTION

## Motivation

The present work is part of the research activity I of the Master “Sustainable concrete construction” completed in the Civil Engineering Faculty of Technical University of Cluj Napoca.

## Objective

Many civil engineering infrastructures and reinforced concrete buildings are in a state of high deterioration due to lack of maintenance, ageing, environmentally induced degradation, poor initial design, accidental events ... Extending the lifetime of existing reinforced concrete (RC) structures is essential to ensure a prosperous economy and a more sustainable future. Therefore, the rehabilitation and repair of these infrastructures is in a great moment, so, it is vital to develop economical and effective strengthening methods for concrete structures. With the advance of time, new materials and techniques have been developed in this area, like strengthening of reinforced concrete structures with carbon fibre composites (CFRP).

The objective of this research it is intended to get a broader idea of this new technique (materials, requirements, execution...) and to follow the recommendations of Bulletin 14 of fib (Europe guide) a several practical examples of reinforcement are presented, in order to better illustrate the guidelines of the guide.

## **2. ADVANTAGES/ DISADVANTAGES**

CFRP materials should not be thought of as a blind replacement of steel in structural intervention applications. The advantages offered by them should be evaluated against potential drawbacks, and final decisions regarding their use should be based on consideration of several factors, including not only mechanical performance aspects, but also constructability and long-term durability.

### **2.1 ADVANTAGES**

The reasons why carbon composites are increasingly used as strengthening materials of reinforced concrete elements may be summarized as follows: immunity to corrosion, low weight (about  $\frac{1}{4}$  of steel), resulting in easier application in confined space, elimination of the need for scaffolding and reduction in labour costs; very high tensile strength (both static and long-term); stiffness which may be tailored to the design requirements; large deformation capacity; and practically unlimited availability in CFRP sizes and CFRP geometry and dimensions.

### **2.2 DISADVANTAGES**

Composites suffer from certain disadvantages too, which are not to be neglected by engineers: contrary to steel, which behaves in an elastoplastic manner, composites in general are linear elastic to failure (although the latter occurs at large strains) without significant yielding or plastic deformation, leading to reduced ductility. Additionally, the cost of materials on a weight basis is several times higher than that for steel (but when cost comparisons are made on strength basis, they become less unfavorable). Moreover, CFRP materials have incompatible thermal expansion coefficients with concrete. Finally, their exposure to high temperatures (e.g. in case of fire) may cause premature degradation and collapse (some epoxy resins start softening at about 45-70 °C).

### 3. APPLICATIONS EXTERNALLY BONDED CFRP REINFORCEMENT (EBR)

Composites have found their way as strengthening materials of reinforced concrete (RC) elements (such as beam, slabs, columns etc.) in thousands of applications worldwide, where conventional strengthening techniques may be problematic.

Steel plates epoxy-bonded to the external surfaces (e.g. tension zones) of beams and slabs. This technique is simple and effective as far as both cost and mechanical performance is concerned, but suffers from several disadvantages: corrosion of the steel plates resulting in bond deterioration; difficulty in manipulating heavy steel plates in tight construction sites; need for scaffolding; and limitation in available plate lengths (Which are required in case of flexural strengthening of long girders), resulting in the need for joints.

Solution: **CFRP strips** (thickness in the order of 1mm) made by pultrusion provides satisfactory solutions to the problems described above.



Jackets (shell= is another common technique for the strengthening of RC structures involves the construction of reinforced concrete (Either cast in-place or shotcrete). Jacketing is clearly quite effective as far as strength, stiffness and ductility is concerned, but it is labour intensive, it often causes disruption of occupancy and it provide RC elements, in many cases, with undesirable weight and stiffness increase.

Solution: The conventional jackets may be replaced with **CFRP fabrics or sheets** (made of fibres in one or at least two different directions, respectively and sometimes pre-impregnated with resin) wrapped around RC elements, thus providing substantial increase in strength (axial, flexural, shear, torsional) and ductility without much affecting the stiffness.



## 4. MATERIALS

The selection of materials or different strengthening systems is a critical process. Every system is unique in the sense that the fibres and the resin components are designed to work together. This implies that a resin system for one strengthening system will not automatically work properly for another.

There are several types of CFRP strengthening systems:

- Wet lay-up systems
- Systems based on prefabricated elements
- Special systems, e.g. automated wrapping, prestressing, etc.

### 4.1 ADHESIVE

The purpose of the adhesive is to provide a shear load path between the concrete surface and the composite material, so that full composite action may develop.

Only the most common type of structural adhesives will be discussed here, namely epoxy adhesive, which is the result of mixing an epoxy resin (polymer) with a hardener. Depending on the application demands, the adhesive may contain fillers, softening inclusions, toughening additives and others.

When using epoxy adhesives there are two different time concepts that need to be taken into consideration. The first is the pot life and the second is the open time. Pot life represents the time one can work with the adhesive after mixing in the resin and the hardener before it starts to harden in the mixture vessel; for an epoxy adhesive, it may vary between a few seconds up to several years. Open time is the time that one can have at his/her disposal after the adhesive has been applied to the adherents and before they are joined together.

Epoxy adhesives have several advantages over other polymers:

- High surface activity and good wetting properties for a variety of substrates.
- May be formulated to have a long open time.
- High cured cohesive strength.
- May be toughened by the inclusion of dispersed rubbery phase.
- Lack of by-products from curing reaction minimizes shrinkage and allows the bonding of large areas with only contact pressure.
- Low shrinkage compared with polyesters, acrylics and vinyl types.
- Low creep and superior strength retention under sustained load.
- Can be made thixotropic for application to vertical surfaces.
- Able to accommodate irregular or thick bond lines.

## 4.2 MATRICES

The matrix for a structural composite material can either be of thermosetting type or of thermoplastic type, with the first being the most common one. The function of the matrix is to protect the fibres against abrasion or environmental corrosion, to bind the fibres together and to distribute the load. The matrix has a strong influence on several mechanical properties of the composite, such as the transverse modulus and strength, the shear properties and the properties in compression.

## 4.3 FIBRES

A great majority of materials are stronger and stiffer in the fibrous form than as a bulky material. A high fibre aspect ratio (length/diameter ratio) permits every effective transfer of load via matrix materials to the fibres, thus enabling full advantage of the properties of the fibres to be taken. Therefore, fibres are very effective and attractive reinforcement materials. Fibres can be manufactured in continuous or discontinuous (chopped) form, but here only continuous fibres are considered. Such fibres have a diameter in the order of 5-20  $\mu\text{m}$ , and can be manufactured as unidirectional or bi-directional reinforcement. The fibres used for strengthening all exhibit a linear elastic behavior up to failure and do not have a pronounced yield plateau as for steel.

## 4.4 CFRP MATERIALS

CFRP materials consist of a large number of small, continuous, directionalized, non-metallic fibres with advanced characteristics, bundled in a resin matrix. Typically, the volume fraction of fibres in CFRPs equals about 50-70% for strips and about 25-35% for sheets. Hence fibres are the principal stress bearing constituents, while the resin transfers stresses among fibres and protects them. Different techniques are used for manufacturing (e.g. pultrusion, hand lay-up), detailed descriptions of which are outside the scope of this bulletin. As externally bonded reinforcement for the strengthening of structures, CFRP materials are made available in various forms.

Material	Elastic modulus (GPa)	Tensile strength (MPa)	Ultimate tensile strain (%)
	$E_f$	$f_f$	$\epsilon_{fu}$
<i>Prefabricated strips</i>			
Low modulus CFRP strips	170	2800	1.6
High modulus CFRP strips	300	1300	0.5
Mild steel	200	400	25*



## 5. TECHNIQUE

The basic technique of strengthening by means of CFRP EBR described here refers to the manual application of CFRP reinforcement to an existing member. The bonding is realized through polymerization of a two-part cold cured bonding agent (normally epoxy-based). All specifications and requirements given in the following sections should be taken into account. The basic technique involves three acting elements, defined as follows:

### Substrate

Is the material type of the existing structure to which the CFRP reinforcement is bonded. Although FRP reinforcement can be applied to different substrates, only a concrete substrate is dealt with in the following sections. The initial conditions of the concrete surface in terms of strength, carbonation, unevenness, imperfections, cracks, type and possible corrosion of internal steel reinforcement, humidity, level of chloride and sulphate ions, etc. should be known.

### Adhesive/Resin

A suitable bonding agent for the CFRP reinforcement that meets all requirements specified. Depending on the type of CFRP reinforcement the bonding agent not only assures the bond between the substrate and the CFRP reinforcement, but also may have to impregnate “wet lay-up” types of CFRP EBR.

### FRP reinforcement

The externally bonded CFRP reinforcement (CFRP EBR) is an advanced composite.

With respect to the application of the CFRP EBR, two major types can be defined. This classification is that used to define the different CFRP EBR systems:

### “Prefab” or “pre-cured” strips or laminates

These CFRP strips are provided as fully cured composites, which have their final shape, strength and stiffness. They are mostly available as thin strips or laminates (thickness about 1.0 to 1.5mm), similar to steel plates. For this type of strip the adhesive provides the bond between the strip and the concrete only.

### “Wet lay-up (hand lay-up)” or “cured in situ” sheets or fabrics

These CFRP materials are available as “Dry fibre”, which means that no resin is inside the CFRP before applying, or “prepreg”, having a very small amount of resin already inside the sheet before applying. In the latter case, the amount of resin is not sufficient for polymerization. For these types of sheets the application of the adhesive is required to both bond the sheet to the concrete and to impregnate the sheet.

An overview of the main characteristics and some typical application aspects of these two types of CFRP EBR are given the table.

	PRE-CURED (PREFAB)	CURED IN SITU (WET LAY-UP)
Shape	Strips or laminates	Sheets or fabrics
Thickness	About 1.0 to 1.5 mm	About 0.1 to 0.5 mm
Use	Simple bonding of the factory made elements with adhesive	Bonding and impregnation of the sheets or fabrics with resin (shaped and cured in-situ)
Typical application aspects	If not pre-shaped only for flat surfaces	Sheets or fabrics
	Thixotropic adhesive for bonding	Low viscosity resin for bonding and impregnation
	Normally 1 layer, multiple layer possible	Often multiple layers
	Stiffness of strip and use of thixotropic adhesive allow for certain surface unevenness	Often a putty is needed to prevent debonding due to unevenness
	Simple in use, higher quality guarantee (prefab system)	Very flexible in use, needs rigorous quality control
	Quality control (wrong application and bad workmanship = loss of composite action between CFRP EBR and substrate/structure, lack of long term integrity of the system, etc.)	

## 6. GENERAL REQUIREMENTS

The technique of external strengthening by means of CFRP EBR can be applied under the following general conditions:

1. On structural elements in a dry or humid environment. Specific requirements are given for these two environmental classes to ensure good bonding. Moreover, in humid environment it should be verified that the internal humidity of the structure (dampness) is not negatively affected by the bonded reinforcement and vice versa. Application of the technique under other environmental conditions (such as under water or on a substrate continuously saturated with water), should be related to a special study of the bond behavior. Even if the adhesive ensures good bond conditions in humid environment, the dampness of the structure can still have other negative influences, such a risk of internal steel corrosion (involving spalling of the concrete cover), susceptibility to freeze/thaw damage of the concrete, etc.

2. The temperature of the strengthened parts of the structure under normal service conditions should not exceed a fraction of the glass transition temperature  $T_g$  of the adhesive or resin. According to prEN 1504-4 (CEN 2001 a), the following conditions applies:  $T_g \geq 45^\circ\text{C}$  or the maximum shade air temperature in service  $+20^\circ\text{C}$ , whichever is the higher.

3. Careful consideration should be given to aspects such as: high differential temperatures between the CFRP EBR and the substrate under service conditions, fire protection, protection against UV, possible occurrence of damage, vandalism, etc.

4. The application of the CFRP EBR for strengthening of structures does normally not confine or arrest defects or potential damage mechanism. Also the concrete should be sound.

5. The strengthening technique should be performed according to the specifications and requirements given in the following sections. In addition, the instructions given by the manufacturer of adhesives, resins and CFRP materials should be taken into account. In conjunction herewith, all necessary design drawings should be available. All information used for the design (including basic assumptions) should be confirmed based on the quality control tests. The application of the CFRP EBR should be performed by qualified and trained workers (preferably, the CFRP EBR system and the operators should be certified by an independent certification body).

## 7. PRACTICAL EXECUTION

The application of the CFRP EBR should be performed in accordance with the following procedure. In addition, any special specifications given by the manufacturers of adhesives and CFRP reinforcement should be followed provided that they are not at variance with these specifications unless backed up by adequate research data. The procedure for the practical application of the externally bonded CFRP reinforcement is depending on the type CFRP EBR, “prefab” or “wet lay-up”

### 7.1 REPAIR

The CFRP EBR generally does not stop existing problems such as steel corrosion, water leakage, high chloride values, etc. Potential damage mechanism such as the risk of steel corrosion in the existing member should be sufficiently low and the concrete should be sound. These aspects should be verified. The following aspects should be considered:

- The minimum concrete tensile strength should be greater than  $1.5 \text{ N/mm}^2$ . If the deteriorated or damaged concrete has reached a depth that no longer allows shallow surface repair, replacement of the concrete should be considered.
- Although the external reinforcement may act as a (partial) replacement of the steel reinforcement, corrosion should be stopped to avoid damage to the concrete due to expansive rust. This damage may result in decreased bond strength and an increased susceptibility to freeze-taw action. Repair or to start protection is needed if the steel is already corroded or is likely to start corroding. With respect to the latter the carbonation depth and chloride content may need to be verified. Generally, chloride concentrations larger than 0.3% by weight of cement are assumed as dangerous.
- To reduce the risk of reinforcing steel corrosion, to solve leakage problems, to avoid weak bond strength at horizontal cracks, etc., wide cracks may need sealing by means of injection. Any cracks (or construction joints) wider than 0.2mm or liable to leakage should be injected by suitable compatible low viscosity resin to fill and seal the cracks. Also, repair of porous concrete and joints to restore water retaining may be of relevance.

### 7.2 PREPARATION SURFACE

#### Concrete substrate

It is important that the preparation of the concrete substrate is carried out well to provide an adequate bond with the adhesive.

1. The substrate should be roughened and made laitance and contamination free, in such a preferably by means of high pressure blasting (sand, grit, water jet blasting) or grinding. In the case of blasting the concrete surface should resemble

coarse sandpaper with minor exposure of aggregates. Most of the wet lay-up systems require a smoother surface so that in this case grinding may be most appropriate or so that the application of a put that may compromise the quality of the outermost concrete should not be allowed (unless the reduce tensile strength and bond characteristics are taken into account in the design). Regardless of the method, execution should not damage the concrete.

2. The concrete should be sound and free from serious imperfections (grind nests, steel corrosion, wide cracks, etc.) and potential damage mechanism.

3. The unevenness that can be allowed depends on the type of CFRP EBR. Strips (or laminates) already have their final stiffness before application and are applied with a high viscosity thixotropic adhesive. In this way, they are less sensitive to unevenness. Fabrics and sheets are very flexible and will follow any unevenness. As a result, the implications of the concrete unevenness are more important for fabrics and sheets.

4. Because the important limitations concerning concrete unevenness, often a putty is supplied together with the resin for wet lay-up systems. This putty shall meet all requirements concerning concrete repair products. The application should be according to the specifications of the manufacturer and the compatibility of the putty and resin for sheet application should be proved.

5. The prepared surface should be dust free before further application of the strengthening technique. This is achieved by cleaning by means of vacuum or oil free compressed air.

6. The prepared surface should normally be surface dry. The allowable surface moisture content is given by the manufacturer. It is noted that certain adhesives can be applied in humid environment. The temperature at the concrete surface should exceed the actual dew point (which also depends on the air humidity) with 5°C. If not, artificial heating and dehumidifiers may be required.

7. The concrete surface shall be marked where the CFRP EBR has to be applied. For complicated strengthening lay-outs the pre-cut CFRP shall be applied temporarily to the concrete. For the application of sheets or fabrics around sharp edges, corners shall be rounded with a radius as specified on the design drawings. If mechanical anchorages are to be provided, all necessary preparations should be performed in an adequate way.

8. Application of a primer is normally not necessary. However, if specified by the manufacturer of the adhesive a primer shall be used according to the specifications given by the manufacturer. A primer may also be specified before applying putty, which is often used to ensure concrete surface evenness.

### CFRP EBR

- The strips and laminates (prefab type) should be supplied to site at specified width and cut to the necessary length as specified on the design drawings. They should be free from any contamination like oil, dust, carbon dust, release agents, etc. For strips provided with an in-built peel ply, to ensure a clean surface, the ply should be removed immediately before application and the surface must not be touched by hand again. If the strips are provided without a peel ply but with a surface ready for bonding, handling should be with extra care. Other strips usually require abrading and wiping clean before use, to obtain a satisfactory surface to which to bond. This should be performed as specified by the manufacturer. The strips should be handled with clean gloves and under dry conditions. They have to be verified for possible damage resulting from transportation, handling or incorrect cutting. The strips and laminates shall be free from unintended curves, bows, wraps, undulations or twists.
- Sheets or fabrics (wet lay-up type). They are cut to the necessary plan-dimensions as specified on the design drawings. They should be kept free from any contamination and checked for possible damage resulting from transportation, handling or wrong cutting. Protecting foils or in-built peel-ply should only be removed just before application. Handling and preparation precautions provided by the manufacturer should be followed. They have to be verified for possible damage resulting from transportation, handling or incorrect cutting. The sheets and fabrics shall be free from wraps, twists or fibre misalignments.

### **7.3 CFRP EBR APPLICATION**

The application will depend on the type of CFRP EBR. For strips and laminates (prefab type) the adhesive ensures bonding only. Often a high viscosity thixotropic adhesive is applied. For sheets and fabrics (wet lay-up type) the resin ensures both bonding and impregnation, which calls for a low viscosity material. The viscosity should still be sufficient to apply the sheets at the soffits of members. The application of the CFRP EBR is performed according to the specifications given in this section and applying appropriate quality control measures. In addition, the information provided by the manufacturer in terms of allowable temperatures and relative humidity, mixing ration, mixing time, pot life, open time, shelf life, provisions given by the safety data sheet, provisions concerning the environmental impact, curing duration, etc. should be taken into account.

The ambient temperature and relative humidity should be within the limits specified by the adhesive or resin manufacturer. The application should be completed within 80% of the pot life (adhesive application) and open time (time for making the joint) of the adhesive at the prevailing temperature.

After application and curing, the CFRP should be essentially straight (concave surface may result in CFRP peeling). Expressed as the depth of the surface variation with respect to a straight base length of 0.3m, the unevenness should be limited to 4mm for “prefab” systems and 2 mm for “wet lay-up” systems.

### “Prefab” type (strips or laminates)

The adhesive is applied as a thin layer to the concrete immediately after mixing. The adhesive is applied to the CFRP sheet in a dome shape (100mm plate width: maximum height about 5 mm), having slightly more thickness along the centre line of the plate. This reduces the risk of forming voids when the strip is applied. The strip is offered to the concrete surface, applying pressure by means of a rubber roller to ensure intimate contact with the concrete. The extra adhesive should be squeezed out along the sides. Also, the pressure is applied in such a way that no voids are formed (going from the centre to the outer). The final bond line should be of equal thickness along the strip and should correspond to a minimum adhesive thickness of 1.5 to 2.0 mm. Normally, the strips are applied in one layer.

Alternatively, instead of applying the adhesive to both the concrete and the strip, adequate results have been reported when applying the adhesive only to one surface. If backed up by adequate research data, this or other alternatives are allowed.

At crossings, the change in thickness of the adhesive should be gradually applied so that the requirements are met.

If masking tape is put on the concrete either side of the strip before gluing, the adhesive surplus can be removed more easily.

Normally, no external pressing devices need to be applied during curing.

### “Wet lay-up” type (sheets or fabrics)

To achieve the required evenness of the concrete surface it will often be specified to apply (a primer and) a putty. This shall be done in accordance with the specifications given by the manufacturer. Next, a low viscosity resin is applied to the concrete (or putty) with sufficient thickness (however the adhesive should remain as an even thickness), by means of a roller brush or a toothed trowel. This process is known as “undercoating”. Then, the fabric or sheet is applied by pressing it manually onto the adhesive in such a way that it is stretched without introduction of voids. Impregnation and further pressing of the sheet is performed by applying adhesive on top of the fabric or sheet (after removal of the paper backing, if present) with a roller brush. This is called “overcoating”. The final bond line should be of an even thickness along the sheet.

Alternative to the above procedure and to increase the level of quality insurance, the fabric or sheet can be impregnated with the resin in a saturator machine. This enables to saturate the CFRP EBR at a better-controlled resin rate and with a more uniform thickness. The wetted fabric or sheet is applied to the sealed substrate.

Often multiple layers will be applied. Unless otherwise specified, this may be done before the previous layer has cured. Normally, no pressing devices need to be applied during curing.

## 7.4 FINISHING

Some form of finishing may be required for aesthetic purpose. In terms of fire protection, possible occurrence of damage, protection against ultra violet radiation, a finishing layer can be crucial to the long-term integrity of the strengthened structure. Different types of finishing layers can be provided such as painting, shot concrete or fire protection panels. These finishing layers should be applied according to the specifications given by the manufacturer. The compatibility between the externally bonded reinforcement and the finishing layer should be proved. If finishing layers or toppings involve heating, this should not damage the bond integrity.

## 7.5 PRACTICAL EXAMPLE

Thanks to MAPEI, in class we had a theoretical and practical lesson about CFRP materials to get more information about this material, know how its works, how is its execution and all types of doubts.





### Tools needed:

- Weighing machine.
- Electric mixer.
- Little roller.
- Scraper.
- Brush.
- Scissors or cutter.
- Bucket.

### Necessary materials:

- 2 little beams of concrete.
- 2 prismatic specimens of concrete.
- 2 cylindrical specimens of concrete.
- MAPEI Adesilex PG1 (epoxy resin).
- MAPEI MAPE Wrap Primer 1 (epoxydic hardener).
- MAPEI Carboplate (CFRP strip).
- MAPEI C UNI -AX (CFRP fabric).
- Sand.

### Steps to follow in the execution

For the concrete beams we used the strips and the fabrics CFRP. For the prismatic specimen only the strips and for the cylindrical specimen only the fabrics because is better due its shape.

When we have a repaired and cleaned surface of the concrete member, we start with the mixing preparation. In a bucket, we mix with the electrical mixer the epoxy resin and the epoxydic hardener in the correct proportion to create the adhesive. Then, we spread with the brush the adhesive in the area where will be the CFRP. The next step is put the CFRP strip over the beam, and then, the same with the CFRP fabric.

We roller over CFRP to get a perfect adhesion to the concrete element. The next step is cover the CFRP with the adhesive and sprinkle sand over it. If we would want put another layer, only we would must repeat the previous steps.

### Pictures

Cutting the CFRP fabric in the needed dimension.



Spreading the adhesive.



Putting the CFRP strip.



Checking if the CFRP strip has good adhesion



Cover the CFRP with more adhesive



Sprinkling sand over the CFRP.



The same procedure with the specimens.



## 8. QUALITY CONTROL

The quality control specifications given in this section only concern the CFRP EBR and its application. For specifications concerning concrete repair techniques and steel corrosion protection techniques, reference is made to corresponding guidelines.

### 8.1 STRENGTH MATERIALS

The adhesives and CFRP EBR used shall be characterized according to standard test methods. The properties should be provided by the manufacturer, who should be able to prove that the tests were performed by an independent laboratory and according to the specified test methods. Furthermore, the manufacturer should guarantee that there is sufficient quality control during production.

If possible, the characterization of materials (according to standard test methods and performed by independent laboratories) should be supervised by a certification body. In this way the products can be certified.

Although standard test methods specifically developed for the characterization of CFRP EBR systems are scarce, in the following reference is made to a number of tests that can be used. More specific or alternative test methods and procedures may become available in the future. In addition to the test methods, some requirements are mentioned.

- **Bonding agent**

A two-part cold cure adhesive, comprising resin and hardener, is used for CFRP bonding. The adhesive is normally epoxy based and similar to the epoxies used for steel plate bonding. Long term experience with epoxy adhesives, including durability, over a period of about thirty years in civil engineering proves their suitability. Other adhesives may be used as long as they provide equivalent performance and that their long-term durability is acceptable.

#### Physical properties

- Viscosity and thixotropy. The polymer bonding agent shall be capable of being applied readily, after mixing according to the manufacturer's instructions, in layer corresponding with the specified adhesive application and with the specified minimum thickness of the final bond line, providing through wetting of the adherents. The viscosity of the adhesive should be optimized with respect to the intended use and will differ considerably for "prefab" and "wet lay-up" types of CFRP EBR. Thixotropy may be required if a high viscosity is wanted with yet sufficient wetting ability during spreading. Different grades of an adhesive, each applicable within a certain working temperature range, may be made available. The suitability for application to vertical surfaces,

soffits and horizontal surfaces is verified according to EN 1799 (CEN 1998 a).

- Curing conditions and shrinkage. The adhesive shall be capable of curing to the required strength under the most extreme conditions with respect to temperature and humidity. These conditions are specified by the manufacturer. A maximum temperature may be specified in relation with the pot (or workable) life and the viscosity. The minimum temperature at which curing is still possible generally equals 5°C. The maximum relative humidity, above which insufficient adhesion is obtained, often equals 80%. On curing, the adhesive shall undergo negligible shrinkage, meaning less than 0.1% determined according to prEN 12617-3 (CEN 2001 b).
- Pot life, open time and shelf life. The mixed adhesive, before application to the prepared surface, shall have a pot life in excess of 40 minutes at 20°C (or at the typical application temperature). The pot (or workable) life is determined according to prEN 14022 (CEN 2001c), The time after application of the adhesive (open time), within which the joint can be made shall exceed at 5-25°C. Adhesives shall not be used if their shelf life, pot life or open time has been exceeded.
- Glass transition temperature. The heat distortion temperature (often characterized by the glass transition temperature  $T_g$ ) should be sufficiently large with respect to the service temperature:  $T_g \geq 45^\circ\text{C}$  or the maximum shade air temperature in service  $+20^\circ\text{C}$ , whichever is the higher. The glass transition temperature is determined according to prEN 12614 (CEN 2001 d).
- Moisture resistance. Moisture transport through the adhesive should be minimized. The maximum water absorption after immersion in water, according to prEN 13580 (CEN 2001 e), shall not exceed 3% by weight.
- Filler properties. Fillers used with the adhesive shall be an electrically non-conductive material, be highly moisture resistant, be able to withstand temperatures up to 120°C without degradation and have a maximum particle size of 0.5mm.

#### Mechanical short-term properties of the cured adhesive:

- Modulus of elasticity in bending (flexural modulus). The modulus of elasticity shall be determined according to ISO 178 (ISO 1993 b) and should be within the 2000-15000 N/mm<sup>2</sup> range. The lower limit relates to a restriction of creep, the upper to minimize stress concentrations.
- Shear strength. A minimum value of 12N/mm<sup>2</sup> at 20°C is required. The shear strength is determined according to EN 12188 (CEN 1999 b).
- Adhesion strength. The adhesion strength of the bonding agent, determined according to EN 12188 (CEN 1999 b) should be larger than 15 N/mm<sup>2</sup> at 20°C
- Compressive strength. The compressive strength shall be determined according to EN 12190 (CEN 1998 b)

#### Durability and long-term properties of the cured adhesive

The durability shall be proved based on laboratory accelerated durability testing or based on long term experience (over a period of at least 15 years) in conditions similar to proposed use. On durability, fatigue and creep under sustained

load testing, reference is made to prEN 13584-1 (CEN 2001 f), prEN 13733-1 (CEN 2001 g), prEN 13894-1 (CEN 2001 h), prEN 13894-2 (CEN 2001 i). These test methods may need to be varied with respect to FRP EBR.

- **CFRP EBR**

Type of CFRP EBR and geometrical characteristics:

The type of CFRP EBR and dimensional characteristics should be specified, in terms of: “prefab” type (plates or strips) and “Cured in-situ” (sheets or fabrics), type(s) of fibre, resin type, fibre directions, width, length and nominal thickness.

The definition of nominal thickness of the CFRP should be clearly indicated. Generally, reference is made to the global thickness or the dry-fibre thickness. If comparison of mechanical properties is made between strips (or laminates) and sheets (or fabrics), the possible differences in defining the nominal thickness should be borne in mind! The equivalent dry-fibre thickness of sheets with multiple fibre directions is related to the fibre direction.

Physical properties of the CFRP EBR:

- Fibre fraction. The fibre weight fraction, the fibre volume fraction or the fibre weight by unit area for each fibre direction shall be provided.
- Amount of resin for impregnation. The minimum amount of resin, per unit area, to impregnate “cured in-situ” sheets or fabrics should be known.
- Coefficient of thermal expansion. The coefficient of thermal expansion shall be determined according to EN 1770 (CEN 1998c)
- Glass transition temperature. The glass transition temperature of “prefab” CFRP types is typically higher (due to factory processing) than that of the bonding agent. The glass transition temperature of “wet lay-up” systems is determined by the resin used for bonding and impregnation.
- Moisture absorption and chemical stability. As also specified for the bonding agent the moisture absorption of the cured CFRP should be limited. Although CFRP EBR systems generally have good chemical stability, the durability of the system in the environment of subject should be demonstrated.

Mechanical short-term properties of the CFRP:

- Tensile strength, elastic modulus and tensile failure strain. The basic properties of the CFRP sheet shall be determined by tensile testing. The tensile stain, the modulus of elasticity at the origin (tangent modulus), the secant modulus (defined between 20-60% of the ultimate load) and the failure strain shall be determined. No standard test method is available, although reference can be made to test methods for CFRP materials used in other fields: EN ISO 527 (ISO 1997), EN 2561 (CEN 1996), ASTM D3039/D3039M (ASTM 1995). The test results should be given as mean values and standard deviation. For the tensile strength and the E-modulus it should be clearly stated which value for the nominal thickness has been taken in consideration. The properties obtained from tensile testing are

related to the fibre direction. For CFRP sheets with multiple fibre directions the properties should be determined for each fibre direction.

#### Durability and long-term properties

The CFRP should have sufficient resistance against moisture, chemicals and UV radiation or should be protected from it by protecting layers. The creep characteristics of the CFRP may be required for the design.

- **Composite action between CFRP EBR, bonding agent and concrete**

The success of strengthening by means of CFRP EBR will be highly dependent on the quality of the bond between the three acting materials: CFRP EBR, bonding agent and concrete substrate. Good bond behavior should be guaranteed under specified (extreme) application conditions and with regard to the long-term durability.

#### Bond between CFRP EBR, bonding agent and concrete

Different type of bond testing can be carried out. Prior to these test, it is recommended that an applicability test will be performed from which further test specimens for bond testing can be taken. With the applicability test the application of the adhesive under specified conditions is verified. At the same time this test can be used for training.

- Applicability test: The material should have a length of 1m, a width corresponding to normal application conditions and a thickness corresponding to the maximum allowable number of layers. The application is performed on the soffit of the slab, the slab being positioned horizontally at an elevation of about 2m. The execution is performed in climate-controlled room under specified conditions (minimum temperature and maximum relative humidity). All materials shall be put 24 hours in advance under the specified conditions. The concrete slab shall be sufficiently large, depending on the CFRP EBR dimensions, and shall have a minimum thickness of 40mm. The concrete quality should be such that a minimum concrete tensile strength of 3 N/mm<sup>2</sup> is guaranteed. Depending on the envisaged application, the concrete substrate will be either ordinary concrete or repaired concrete. In the latter case, an additional repair mortar layer with thickness 10 mm is applied on the concrete surface. The strengthened slab is cured under the specified conditions for 7 days and additionally during an extra 7 days under laboratory conditions (20±5°C). After execution the CFRP EBR is evaluated in terms of unevenness, thickness of the bond line and voids. In addition, concrete cores diameter 50 mm are drilled for direct tensile testing after 3, 7 days and 14 days to evaluate the bond performance. The above testing conditions refer to “dry environment”. If the adhesive allows for “humid environment” the concrete slab will be immersed in water for 7 days. The application test starts 2 hours after taking the slab out of the water and drying the surface with a towel until surface dry.
- Bond performance in direct tension: The bond performance can be evaluated by means of direct tensile testing of the CFRP EBR/bonding

agent/concrete substrate combination. Test specimens are obtained by taking cores from the applicability test specimen. Tests are performed at 7 days and at 14 days under the specified curing conditions. In addition, for outdoor exposure, durability testing shall be performed. The test method will be according to EN 1542 (CEN 1999 c). The failure should be located inside the concrete.

- Durability testing: The durability will be evaluated based on the bond performance in direct tension, after freeze/thaw or outdoor ageing. The cores taken from the applicability test specimen and cured under the specified conditions are respectively submitted to 10 freeze/thaw test are according to a cycle of 4 hours at -20°C 4 hours at +60°C and 16 hours under water at 20°C. The specimens stored in an outdoor climate are left exposed for minimum 1 year according to prEN 13733 (CEN 2001 g).
- Bond performance in shear: No standard test method is currently available, although this subject is under investigation in a round robin test (TMR ConFibreCrete, fib TG9.3 and ISIS 2001)

#### Quality of adhesive bonded joints

If the CFRP EBR is not available in quasi-continuous lengths or if more layers are applied than the maximum specified by the supplier, adhesive bonded joint testing has to be performed (so-called lap shear strength test). This can be done according to EN 1465 (CEN 1995).

## **8.2 QUALIFICATION WORKERS**

The strengthening technique (including possible preceding repair techniques) shall be performed by qualified and experienced workers. The team chief or work coordinator (foreman) shall be trained and qualified on all aspects of the applied techniques and shall be present during work at all times.

## **8.3 QUALITY CONTROL PRACTICAL EXECUTION**

The products shall be provided with the following information:

- General data (such as: name, type and function of the product, product components, name and address of the manufacturer, batch number and expiry date).
- On request, data concerning the material properties according to standard test methods.
- Information concerning handling, transportation and storage (such as: pot or workable life, shelf life, mixing ratios, mixing requirements, storage conditions, application conditions, application guidance, cleaning agents, curing time in between primer and putty applications, etc.)
- Safety data (such as: toxicity, inflammability, environmental impact, etc).

The components of the bonding agents (primer, putty, adhesive) shall be provided separately and pre-dosed according to the mixing ratio. Preferably, one



component (in normal-sized packing) is added to a second component (in oversized packing). It should be verified that no material remains in the packing of the first component. The mixing speed should be sufficiently low to avoid formation of air bubbles. Mixing should be continued until a homogeneous colour is obtained. It should be verified that no badly mixed zones are remaining.

Special care should be taken in avoiding any type of damage to the CFRP EBR during transportation, storage and handling. Cutting of CFRP EBR to length according to the design drawings is allowed on construction site, if cutting is performed according to the specifications of the manufacturer and if no further damage to the sheet is initiated.

#### Quality control of the supplied materials

Representative test samples should be taken from the supplied materials. The number of control tests will depend on the importance of the job (the total area to be strengthened by CFRP EBR bonding, the difficulty in strengthening lay-out and whether or not the materials are supplied in total). At least 3 tensile tests of the CFRP EBR and 6 bonding agent compression tests should be performed, unless the products are certified by independent certification bodies.

#### Quality control on the application conditions.

The suitability of the concrete substrate for bonding should be verified before and after repair techniques. The concrete quality is verified based on the tensile strength of the concrete surface, by means of pull-off testing according to EN 1542 (CEN 1999 c). The obtained strength should be equal to the tensile strength of the bulk concrete (locus of the failure to be totally within the concrete), unless taken into consideration in the design. The minimum strength should be 1.5 N/mm<sup>2</sup>. Furthermore, air humidity, air temperature and surface moisture and temperature of the concrete substrate are measured to evaluate the environmental conditions (“dry” or “humid” conditions, dew point, temperature limitations for use of the adhesive, maximum allowable relative humidity, etc.)

#### Quality control on the application process

During the execution and at finishing quality control verifications are performed to ensure good quality of the strengthening:

- Quality control to verify that the proper execution procedure is followed and that the CFRP EBR is applied in the given direction and with proper amounts of fibres and polymer. It has also to be ensured that proper mixing and manipulation of resin and correct surface preparation takes place.
- Verification of satisfactory unevenness of the CFRP EBR and bond line thickness after execution.

## 8.4 BOND QUALITY AFTER PRACTICAL EXECUTION

Further quality tests of the bond interface (presence of voids or defects, bond strength) may not be necessary when the contribution of the bond interface in stress transmission between the CFRP and the concrete is negligible (i.e. columns wrapping). In most cases however, it may be required to perform quality control of the bond interface.

Quality control of bonding can be achieved through Non-Destructive Testing (NDT) and Partially-Destructive Testing (PDT). NDT are to be preferred for testing critical areas of the strengthening where CFRP contribution is assumed to be fundamental, and in general in all that all situations where the strengthened surface is small in comparison with the area damaged by PDT. To aid the quality control, during execution separate test areas may be foreseen. These areas are to be executed together with the actual CFRP EBR application and under exactly the same conditions.

Different tests can be performed as outlined in the following. It is recommended that at least 3 bond tests are performed at 3 days and/or at 7 days and that critical bond zones are scanned for the presence of voids.

### Partially destructive techniques

- Surface adherence pull-off test: Partial coring is performed in the CFRP EBR 5mm into the concrete. A disc is glued and after hardening pulled off in direct tension. The test is performed according to EN 1542 (CEN 1999 c).
- Surface adherence shear test: IF a CFRP test strip has been glued on the concrete surface close to an edge and extending from it, the extending part of the strip can be gripped and submitted to tension until rupture at the strip-concrete interface occurs. In this type of test the bond interface is submitted to pure shear.
- Surface adherence torque test: A ring disc is glued on the CFRP strengthening surface and partial coring is performed at the outer and inner diameter, extending 5mm into the concrete. After hardening the disc is twisted off. In this type of the test the bond interface is submitted to torque.

### Non destructive techniques.

To verify that no large voids in the adhesive are present the following tests may apply

- Tapping: Normally voids can be detected by means of “tapping” The boning surface with a steel stick with diameter 5mm and with a rounded tip.
- Ultrasonic pulsed echo techniques: A high frequency ultrasonic beam is used to scan CFRP-concrete interface and bonding defects are located through echoes generated by acoustic impedance mismatching. Echo amplitude techniques are to be preferred to time-of-flight ones. Effectiveness of the ultrasonic echo technique is limited to “gaseous” defects such as air bubbles or gas film detachments, is critical under CFRP

loading conditions and requires smooth CFRP surfaces and well-experienced personnel. CFRP areas next to edges or with small bending radius cannot be successfully tested with this technique. Since technique is time consuming, its strongly recommended only for the strengthening areas where bonding is critical.

- Ultrasonic transparency techniques: A low frequency ultrasonic beam is used to scan paths orthogonal to the strengthened surface where one of the transducer is placed, while another transducer is located on the opposite face of the concrete element. Time-of-flight and wave attenuation are recorded. The technique is applicable only when both are strengthened and the opposite surface are easily accessible and is time consuming. Its use should be recommended only in particular situations.
- Thermography: Thermography NDT in direct dynamic condition has been successfully applied to bonding evaluation. Testing should take place with the specimen at thermal equilibrium, applying a homogeneous heating (or cooling) to the CFRP free surface and system. Defects are located as hot (or cool) spots due to different thermal properties of degraded bonding. Severe limitations of the technique arise with thick overlays and with CFRP materials with a high thermal conductivity, such as CFRP. Although thermography is strongly recommended, great care has to be used in calculating the smallest detectable defect in order to ensure it is not critical for crack propagation. Calculation can be performed using thermodynamic model of the strengthening.
- Other dynamic techniques: At present some other dynamic techniques based on impact spectrum analysis and on surface acoustic wave propagation are under evaluation.

#### Corrective actions for bonding defects.

If considerable voids are present, the CFRP should be stripped off and new CFRP reinforcements should be glued. Alternatively, they may be injected with a compatible resin, according to a procedure agreed on by all parties. Herewith, the reduction in CFRP sheet section, the localized injection pressure, and all other negative influences resulting from the injection should be taken into consideration to see if still all design requirements are met.

### **8.5 IN-SERVICE INSPECTION AND MAINTENANCE**

A manual procedures for inspection and maintenance for the repaired and strengthened structure should be prepared for the maintaining authority.

## 9. EXAMPLES

### 9.1 BEAM SIMPLY SUPPORT – FLEXURAL STRENGTHENING

In this section we are going to propose a practical case for the design of the reinforcement by means of CFRP system of a bending beam. The calculation of the reinforcement is going to focus expressly on the development of the working procedure to meet the resistance required by the beam subjected to bending stresses.

We are going to study the practical case of a reinforced concrete beam of rectangular section, 30x50 cm, and 5.00 m. of length, subjected to uniformly distributed permanent loads, including its own, ( $G = 18 \text{ KN / m}$ ) and variable ( $Q = 15 \text{ KN / m}$ ). It is a simply support beam located inside a building. From a project of change of use in the building, it has been calculated that the variable loads that act on the beam increase by 50% the previous ones. Our job is to check if the beam resistance can withstand this increase in loads or, on the other hand, we need to reinforce the beam. If reinforced, the system composed of carbon fiber laminates externally adhered to the traction face of the concrete beam has been proposed, so that the free height of the floor is not reduced.

#### Materials

Concrete: C25/30

$f_{ck} = 25\text{MPa}$ ;  $f_{cd} = f_{ck} / \gamma_c = 25 / 1.5 = 16.67\text{MPa}$ ;  $E_c = 31\text{GPa}$

Steel: S500

$f_{yk} = 500\text{MPa}$ ;  $f_{yd} = f_{yk} / \gamma_y = 500 / 1.15 = 434.78\text{MPa}$ ;  $E_s = 205\text{GPa}$

#### Dimensions

$L = 5.00 \text{ m}$ ;  $h = 0.4\text{m}$ ;  $b = 0.25\text{m}$

$c_{nom} = 0.03\text{m}$ ;  $d = h - c_{nom} = 0.4\text{m} - 0.03\text{m} \rightarrow d = 0.37\text{m}$

#### Initial loads

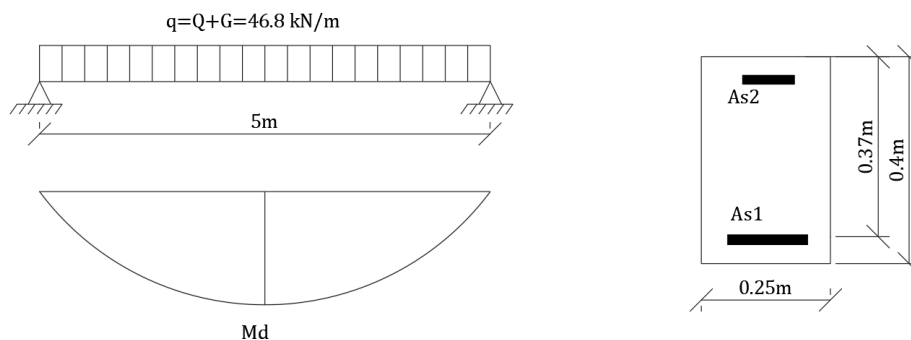
$G = 18\text{kN/m}$  (self-weight included)  $\cdot 1.35$  ( $\gamma_G$ ) =  $24.3\text{kN/m}$

$Q = 15\text{kN/m} \cdot 1.5$  ( $\gamma_Q$ ) =  $22.5\text{kN/m}$

$q = G + Q = 24.3 + 22.5 \rightarrow q = 46.8\text{kN/m}$

#### Initial situation

First, we will calculate the reinforced of the beam to support the initial loads that act on it.



$$M_d = \frac{q \cdot L^2}{8} = \frac{46.8 \cdot 5^2}{8} = 146.25 \text{ kNm}$$

$$\mu = \frac{M_d}{b \cdot d^2 \cdot f_{cd}} = \frac{146.25 \cdot 10^3}{250 \cdot 370 \cdot 16.67} = 0.2563 < 0.372 \text{ (}\mu \text{ limit S500)}$$

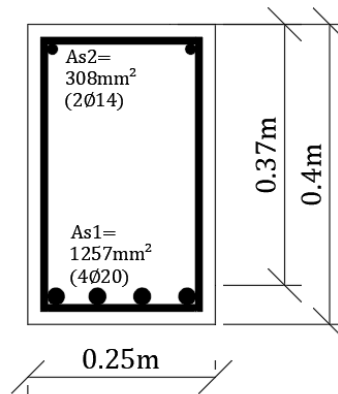
$$\omega = 1 - \sqrt{1 - 2 \cdot \mu} = 1 - \sqrt{1 - 2 \cdot 0.372} = 0.30191$$

$$A_{s1} = \omega \cdot b \cdot d \cdot \frac{f_{cd}}{f_{yd}} = 0.30191 \cdot 250 \cdot 370 \cdot \frac{16.67}{434.78} = 1070.76 \text{ mm}^2$$

$$4\emptyset 20; A_{s1\text{eff}} = 1257 \text{ mm}^2$$

$$A_{s2} = 0.5 \cdot \frac{f_{tm}}{f_{yk}} \cdot b \cdot d = 0.5 \cdot \frac{2.6}{500} \cdot 250 \cdot 370 = 240.5 \text{ mm}^2$$

$$2\emptyset 14; A_{s2\text{eff}} = 308 \text{ mm}^2 \text{ (Geometrical reinforcement)}$$



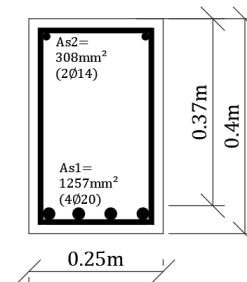
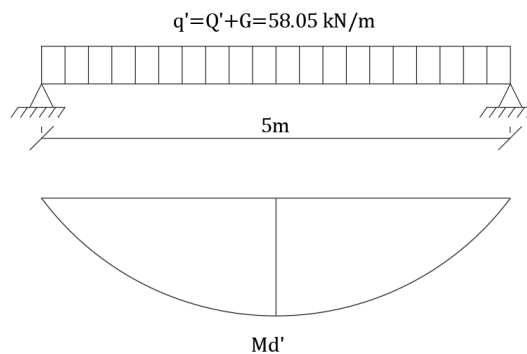
Change needs of the building: Variable loads +50% ( $Q' = Q \cdot 1.5$ )

### New loads

$$G = 18 \text{ kN/m} \cdot 1.35 (\gamma_G) = 24.3 \text{ kN/m}$$

$$Q' = 15 \text{ kN/m} \cdot 1.5 = 22.5 \text{ kN/m} \cdot 1.5 (\gamma_Q) = 33.75 \text{ kN/m}$$

$$q' = G + Q' = 24.3 + 33.75 \rightarrow q' = 58.05 \text{ kN/m}$$



$$M'_d = \frac{q' \cdot L^2}{8} = \frac{58.05 \cdot 5^2}{8} = 181.4 \text{ kNm}$$

Once the resistant design moment related to the change of use has been calculated, we will check whether the reinforcement of the beam is enough to support the new request or if on the contrary it is necessary to reinforce.

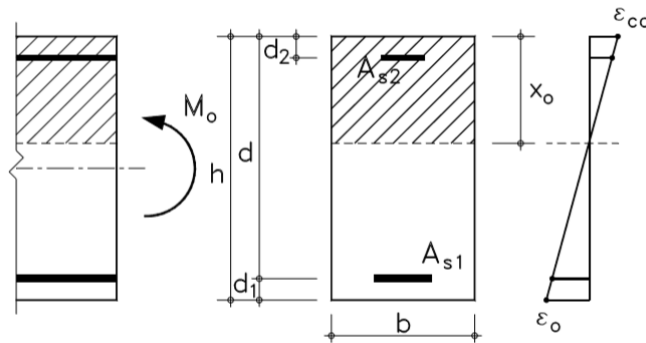
If  $M'd \geq Mu$  is necessary the strengthening

$$Mu = As1 \cdot f_{yd} \cdot d \cdot \left[ 1 - 0.5 \cdot \left( \frac{As1 \cdot f_{yd}}{f_{cd} \cdot b \cdot d} \right) \right]$$

$$Mu = 1257 \cdot 434.78 \cdot 370 \cdot \left[ 1 - 0.5 \cdot \left( \frac{1257 \cdot 434.78}{16.67 \cdot 250 \cdot 370} \right) \right] = 166377105.9 \text{ N/mm}$$

$$M'd = 181.4 \text{ kNm} > Mu = 166.37 \text{ kN/m}$$

The section of the beam does not support the new moment of solicitation since the last moment is less than the moment that it must resist as a result of applying the overload. The section has to be reinforced and the application of the reinforcement by means of carbon fiber laminates (CFRP) applied on the underside of the beam is opted for.



Neutral axis depth =  $Xo$

$$\frac{1}{2} \cdot b \cdot Xo^2 + (\alpha_s - 1) \cdot As2 \cdot (Xo - d_2) = \alpha_s \cdot As1 \cdot (d - Xo)$$

$$\alpha_s = \frac{E_s}{E_c} = \frac{205}{31} = 6.61$$

$$Mo = \frac{G \cdot L^2}{8} = \frac{18 \cdot 5^2}{8} = 56.25 \text{ kNm (No load safety factors are applied)}$$

$$\frac{1}{2} \cdot 250 \cdot Xo^2 + (6.61 - 1) \cdot 308 \cdot (Xo - 30) = 6.61 \cdot 1257 \cdot (370 - Xo)$$

$$125 \cdot Xo^2 + 1727 \cdot Xo - 51836.4 = 3074244 - 8308.77 \cdot Xo$$

$$125 \cdot Xo^2 + 10035.77 \cdot Xo - 3126080.4 = 0$$

$$Xo = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \rightarrow \boxed{Xo = 123.01 \text{ mm}}$$

Next, we calculate the moment of inertia of the cracked section that will be necessary to calculate the deformation of the concrete  $\epsilon_o$ :

$I_{o2}$  = Moment of inertia of the transformed cracked section

$$I_{o2} = \frac{b \cdot X_o^3}{3} + (\alpha_s - 1) \cdot A_{s2} \cdot (X_o - d_2)^2 + \alpha_s \cdot A_{s1} \cdot (d - X_o)^2$$

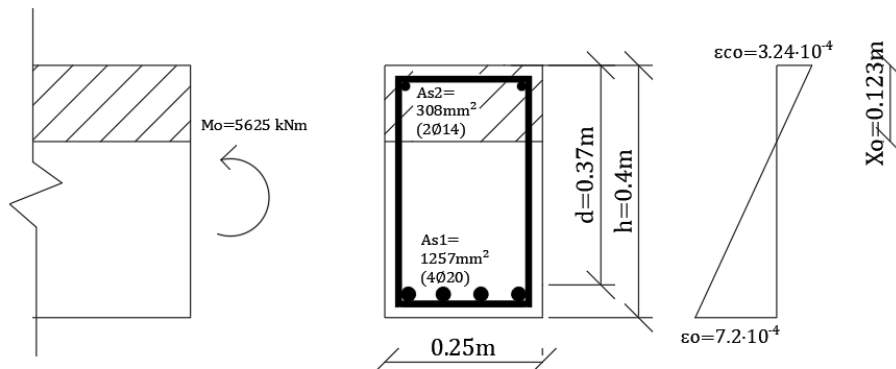
$$I_{o2} = \frac{250 \cdot 123^3}{3} + (6.61 - 1) \cdot 308 \cdot (123 - 30)^2 + 6.61 \cdot 1257 \cdot (370 - 123)^2$$

$$I_{o2} = 155072250 + 14944434.12 + 506909748.9 \rightarrow I_{o2} = 676926433.1 \text{ mm}^4$$

$$\epsilon_{co} = \frac{M_o \cdot X_o}{E_c \cdot I_{o2}} = \frac{56250000 \cdot 123}{31000 \cdot 676926433.1} \rightarrow \epsilon_{co} = 3.24 \cdot 10^{-4}$$

Finally, by deformation compatibility, we calculate the deformation of the concrete in the extreme fiber to tension:

$$\epsilon_o = \epsilon_{co} \cdot \frac{h - X_o}{X_o} = 3.29 \cdot 10^{-4} \cdot \frac{400 - 123}{123} \rightarrow \epsilon_o = 7.4 \cdot 10^{-4}$$



### Analysis of Ultimate Limit State

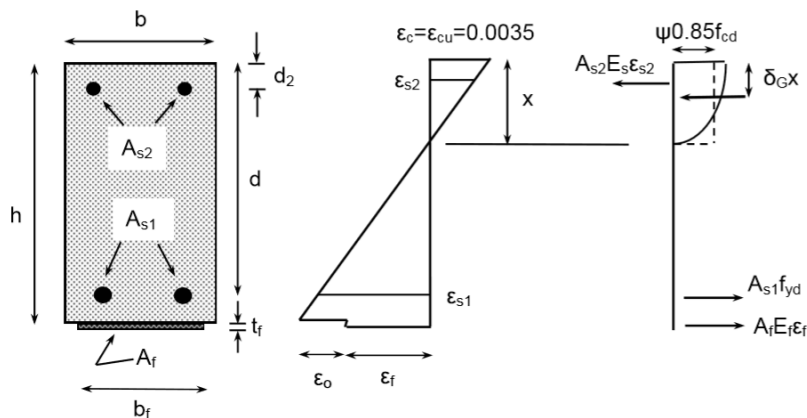
According to FIB, the desirable working principle is to reach the elastic limit of the steel followed by the crushing of the concrete and thus we will perform the calculation. We start with the moment after the overload, with the increase of variable loads.

CFRP that we use: CARBOPLATE E170

Thickness:  $t_f = 1.4 \text{ mm}$ ; Width:  $b_f = 50 \text{ mm}$ ;

Area:  $A_f = 70 \text{ mm}^2$

$\sigma_{fu} = f_{fu} = 3100 \text{ MPa} = 3.1 \text{ kN/mm}^2$ ;  $\epsilon_{fu} = 0.018$ ;  $E_{fu} = 170 \text{ GPa} = 170 \text{ kN/mm}^2$



$$0.85 \cdot \phi \cdot f_{cd} \cdot b \cdot x + A_{s2} \cdot E_s \cdot \epsilon_{s2} = A_{s1} \cdot f_{yd} + A_f \cdot E_{Fu} \cdot \epsilon_f$$

$$\phi = 0.8$$

To solve the depth of the neutral axis, we will start from an estimated value that we will introduce first in the equations to find the deformations in the compression steel and in the laminate. Calculated these values, we will introduce them into the equation of the neutral axis to find the value of it, which must coincide with the previous one. It is a process of iteration until reaching the value of  $x$  that satisfies the principle in the ultimate limit state of deformation compatibility and internal balance of forces (first we start from  $x=0.2d=0.9m$ ).

$$\begin{aligned}\epsilon_{s2} &= \epsilon_{cu} \cdot \frac{x - d_2}{x} = 0.0035 \cdot \frac{74 - 30}{74} \rightarrow \epsilon_{s2} = 0.002081 \\ E_s \cdot \epsilon_{s2} &< f_{yd} \rightarrow \epsilon_{s2} \cdot E_s = 205000 \cdot 0.002081 = 426.60 < 434.78 = f_{yd} \\ \epsilon_f &= \epsilon_{cu} \cdot \frac{h - x}{x} - \epsilon_0 = 0.0035 \cdot \frac{400 - 74}{74} - 7.4 \cdot 10^{-4} \rightarrow \epsilon_f = 0.01467\end{aligned}$$

We introduce the values in the equation of the neutral axis:

$$\begin{aligned}0.85 \cdot \varphi \cdot f_{cd} \cdot b \cdot x + A_{s2} \cdot E_s \cdot \epsilon_{s2} &= A_{s1} \cdot f_{yd} + A_f \cdot E_{Fu} \cdot \epsilon_f \\ 0.85 \cdot 0.8 \cdot 16.67 \cdot 250 \cdot x + 308 \cdot 426.6 &= 1257 \cdot 434.78 + 70 \cdot 170000 \cdot 0.01467 \\ 2833.9 \cdot x + 133912.24 &= 546518.46 + 174573 \\ 2833.9 \cdot x = 587179.22 \rightarrow x &= \frac{587179.22}{2833.9} \rightarrow x = 207.198 \neq 74\end{aligned}$$

The first depth proposal of the neutral axis is not correct. Let's estimate as second proposal,  $x = 207.198$  mm:

$$\begin{aligned}\epsilon_{s2} &= \epsilon_{cu} \cdot \frac{x - d_2}{x} = 0.0035 \cdot \frac{207.198 - 30}{207.198} \rightarrow \epsilon_{s2} = 0.003 \\ E_s \cdot \epsilon_{s2} &= 205000 \cdot 0.003 = 615 > f_{yd} = 434.78 = \epsilon_{s2} \cdot E_s \\ \epsilon_f &= \epsilon_{cu} \cdot \frac{h - x}{x} - \epsilon_0 = 0.0035 \cdot \frac{400 - 207.198}{207.198} - 7.4 \cdot 10^{-4} \rightarrow \epsilon_f = 0.0025\end{aligned}$$

We introduce the new values in the equation of the neutral axis:

$$\begin{aligned}0.85 \cdot \varphi \cdot f_{cd} \cdot b \cdot x + A_{s2} \cdot E_s \cdot \epsilon_{s2} &= A_{s1} \cdot f_{yd} + A_f \cdot E_{Fu} \cdot \epsilon_f \\ 0.85 \cdot 0.8 \cdot 16.67 \cdot 250 \cdot x + 308 \cdot 434.78 &= 1257 \cdot 434.78 + 70 \cdot 170000 \cdot 0.0025 \\ 2833.9 \cdot x + 133912.24 &= 546518.46 + 29750 \\ x = \frac{442356}{2833.9} \rightarrow x &= 156.09 \neq 207.198\end{aligned}$$

We continue with the process until it matches the previous value:

$$\begin{aligned}\epsilon_f = 0.0047 \rightarrow x &= 165.33 \neq 156.09 \\ \epsilon_f = 0.0042 \rightarrow x &= 163.35 \neq 165.33 \\ \epsilon_f = 0.0043 \rightarrow x &= 163.78 \approx 163.35 \rightarrow \boxed{x = 163.5\text{mm}}\end{aligned}$$

We already have the depth of the neutral axis in ELU, where the compression steel works in a plastic regime and the deformation of the laminate is lower than the limit deformation of the steel (10 ‰). We calculate the design resistant moment:

$$\begin{aligned}M_{rd} &= A_{s1} \cdot f_{yd} \cdot (d - \delta_g \cdot x) + A_f \cdot E_f \cdot \epsilon_f (h - \delta_g \cdot x) + A_{s2} \cdot E_s \cdot \epsilon_{s2} \cdot (\delta_g \cdot x - d_2) \\ \delta_g &= 0.4 \\ M_{rd} &= 1257 \cdot 434.78 \cdot (370 - 0.4 \cdot 163.5) + \\ &+ 70 \cdot 170000 \cdot 0.0043 \cdot (400 - 0.4 \cdot 163.5) + 308 \cdot 434.78 \cdot (0.4 \cdot 163.5 - 30) \\ M_{rd} &= 166469522.9 + 17121482 + 4740493 \rightarrow M_{rd} = 188331498.2\text{Nmm}\end{aligned}$$

$$\boxed{M_{rd} = 188.33\text{kNm} > M'd = 181.4\text{kNm} \text{ is OK}}$$



The resistant moment of the section of the beam with the CFRP laminated reinforcement is greater than the stress to which it is loaded. The 50 mm carbon fiber reinforcement is correct and will be placed on the axis of the lower face of the beam. As indicated in bulletin 14, for simply supported beam, the laminate will be placed over the entire length of the beam, until to a distance of the support no greater than 50 mm.

For the entire procedure to be valid, we check the following equations of the yield strength of the steel working under tension and the deformation of the CFRP laminate, which must be less than the ultimate deformation.

CFRP in the axis of the lower face (side)

$$\varepsilon_{s1} = \varepsilon_{cu} \cdot \frac{d - x}{x} \geq \frac{f_{yd}}{E_s} \rightarrow \varepsilon_{s1} = 0.0035 \cdot \frac{370 - 163.5}{163.5} \geq \frac{434.78}{205000} \rightarrow$$

$$\rightarrow \boxed{\varepsilon_{s1} = 4.42 \cdot 10^{-3} \geq 2.12 \cdot 10^{-3} \text{ is OK}}$$

$$\varepsilon_f = \varepsilon_{cu} \cdot \frac{h - x}{x} - \varepsilon_o \leq \varepsilon_{fud} \rightarrow$$

$$\varepsilon_f = 0.0035 \cdot \frac{400 - 163.5}{163.5} - 7.4 \cdot 10^{-4} \leq 0.018 \rightarrow$$

$$\rightarrow \boxed{\varepsilon_f = 4.32 \cdot 10^{-3} \leq 1.8 \cdot 10^{-2} \text{ is OK}}$$

The last section of this step corresponds to verify that sufficient ductility is obtained. According to the FIB ductility criterion, we must verify that:

$$M_{rd} < 1.2 \cdot M'd$$

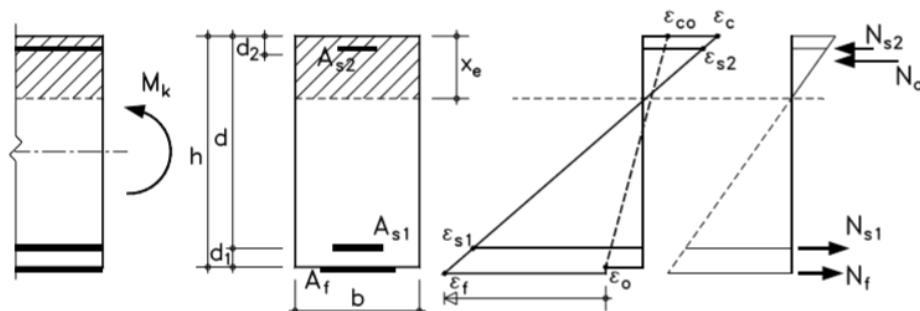
$$188.33 < 1.2 \cdot 181.4 \rightarrow \boxed{188.33 \text{ kNm} < 217.68 \text{ kNm is OK}}$$

$$\xi \leq 0.45 \text{ Concrete c25/30}$$

$$\xi = \frac{x}{d} = \frac{163.5}{370} \rightarrow \boxed{0.44 < 0.45 \text{ is OK}}$$

### Analysis Serviceability Limit State

Calculations to verify the serviceability limit state may be performed according to a linear elastic analysis. Reference will be made to both uncracked (state 1) and cracked sections (state 2). Whereas the neutral axis depth of RC members, according to a linear elastic calculation, is independent from the acting moment, this is no longer the case for a strengthened section as a result of the initial strains before strengthening. Assuming linear elastic material behaviour and that the concrete does not sustain tension, the cracked section analysis can be based on the next figure:



We start with the depth of the neutral axis, by means of balance of forces and compatibility of deformations:

$$\frac{1}{2} \cdot b \cdot X_e^2 + (\alpha_s - 1) \cdot A_{s2} \cdot (x_e - d_2) = \alpha_s \cdot A_{s1} \cdot (d - X_e) + \alpha_f \cdot A_f \cdot \left[ h - \left( 1 + \frac{\epsilon_0}{\epsilon_c} \right) + X_e \right]$$

$$\alpha_f = \frac{E_f}{E_c} = \frac{170000}{31000} = 5.48$$

$$\alpha_s = \frac{E_s}{E_c} = \frac{205000}{31000} = 6.61$$

$$\left( 1 + \frac{\epsilon_0}{\epsilon_c} \right) = 1$$

$$\frac{1}{2} \cdot b \cdot X_e^2 + (\alpha_s - 1) \cdot A_{s2} \cdot (x_e - d_2) = \alpha_s \cdot A_{s1} \cdot (d - X_e) + \alpha_f \cdot A_f \cdot \left[ h - \left( 1 + \frac{\epsilon_0}{\epsilon_c} \right) + X_e \right]$$

$$\frac{1}{2} \cdot 250 \cdot X_e^2 + (6.61 - 1) \cdot 308 \cdot (x_e - 30) = 6.61 \cdot 1257 \cdot (370 - X_e) + 5.48 \cdot 70 \cdot [400 - 1 \cdot X_e]$$

$$125 \cdot X_e^2 + 10420 \cdot X_e - 3279521.3 = 0$$

$$X_e = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \rightarrow X_e = 125.57 \text{ mm}$$

We proceed to calculate the combinations of actions in ELS. For the calculation, we take the acting loads without safety coefficient:

Different loads combinations:

- Under the rare load combination

$$\sum Q_{k,j} + P + Q_{k,1} + \sum_{i>1} \varphi_{0,i} Q_{k,i}$$

$\sum Q_{k,j}$  = characteristic value of permanent load

P = Representative of prestressed load

$Q_{k,1}$  = Characteristic value of variable load 1

$\sum_{i>1} \varphi_{0,i} Q_{k,i}$  = Characteristic value of variable load, reduced.

$$\sum Q_{k,j} + Q_{k,1} = 18 + 22.5 = \frac{40.5 \text{ kN}}{\text{m}}$$

- Under the quasi-permanent load combination

$$\sum Q_{k,j} + P + \sum_{i>1} \varphi_{2,i} Q_{k,i}$$

Our situation only  $\sum Q_{k,j}$  and  $\sum_{i>1} \varphi_{2,i} Q_{k,i}$

(reduced by coefficient  $\varphi_2 = 0.3$ , category A, Table A 1.1 Eurocode 1)

$$\sum Q_{k,j} + \sum_{i>1} \varphi_{2,i} Q_{k,i} = 18 + (22.5 \cdot 0.3) = 24.75 \text{ kN/m}$$

Calculated the neutral axis, we have two values of  $E_c \cdot \epsilon_c$  depending on the combination of actions. We check in this way if it meets the SLS requirement:

- Under rare load combinations

$$M_{k,r} = \frac{q \cdot L^2}{8} = \frac{40.5 \cdot 5^2}{8} = M_{k,r} = 126.56 \text{ kNm}$$

$$E_c \cdot \epsilon_c = \frac{126560000}{\frac{1}{2} \cdot b \cdot X_e \cdot \left( 1.05 \cdot d - \frac{X_e}{3} \right)} = \frac{126560000}{\frac{1}{2} \cdot 250 \cdot 125.57 \cdot \left( 1.05 \cdot 370 - \frac{125.57}{3} \right)}$$

$$E_c \cdot \epsilon_c = \frac{126560000}{6097993.125 - 656992.7} \rightarrow E_c \cdot \epsilon_c = 23.26 \text{ N/mm}^2$$

Stress limitation

$$\sigma_{c,r} = E_c \cdot \epsilon_c \leq 0.6 \cdot f_{ck} \rightarrow \sigma_{c,r} = 23.26 \geq 0.6 \cdot 25 = 15 \text{ is not OK}$$

- Under quasi-permanent load combinations

$$M_{k, qp} = \frac{q \cdot L^2}{8} = \frac{24.75 \cdot 5^2}{8} = M_{k, qp} = 77.34 \text{ kNm}$$

$$E_c \cdot \varepsilon_c = \frac{M_{k, qp}}{\frac{1}{2} \cdot b \cdot X_e \cdot (1.05 \cdot d - \frac{X_e}{3})} = \frac{77340000}{\frac{1}{2} \cdot 250 \cdot 125.57 \cdot (1.05 \cdot 370 - \frac{125.57}{3})}$$

$$\rightarrow E_c \cdot \varepsilon_c = 11.25 \text{ N/mm}^2$$

Stress limitation

$$\sigma_{c, qp} = E_c \cdot \varepsilon_c \leq 0.45 \cdot f_{ck} \rightarrow \sigma_{c, qp} = 14.21 \geq 0.45 \cdot 25 = 11.25 \text{ is not OK}$$

To prevent yielding of the steel at service load, Eurocode 2 specifies:

- Under rare load combination

$$\sigma_s = E_s \cdot \varepsilon_c \cdot \frac{d - X_e}{X_e} \leq 0.8 \cdot f_{yk}$$

$$\varepsilon_c = \frac{\sigma_{c, r}}{E_c} = \frac{23.26}{31000} = 7.5 \cdot 10^{-4}$$

$$\sigma_s = 205000 \cdot (7.5 \cdot 10^{-4}) \cdot \frac{370 - 125.57}{125.57} = 299.28$$

$$\sigma_s = 299.28 \leq 0.8 \cdot f_{yk} = 0.8 \cdot 500 = 400 \text{ is OK}$$

- Under quasi-permanent load combination

$$\sigma_f = E_f \cdot (\varepsilon_c \cdot \frac{h - X_e}{X_e} - \varepsilon_o) \leq \eta \cdot f_{fk}$$

$$\eta = 0.8 \text{ (for CFRP)}$$

$$\varepsilon_c = \frac{\sigma_{c, qp}}{E_c} = \frac{14.21}{31000} = 4.58 \cdot 10^{-4}$$

$$\sigma_f = 170000 \cdot [(4.58 \cdot 10^{-4}) \cdot \frac{400 - 125.57}{125.57} - 7.4 \cdot 10^{-4}] = 44.36 \text{ N/mm}^2$$

$$\sigma_f = 44.36 \leq 0.8 \cdot f_{fk} = 0.8 \cdot 3100 = 2480 \text{ N/mm}^2 \text{ is OK}$$

As can be observed in the result, few deformations are expected in the reinforcement under service load, the creep rupture thereof is not a concern.

## Conclusions

	G	Q	q		Md	
Origin loads (kN/m <sup>2</sup> )	18	15	46,8	Origin moment	146,25	Mu=166,4
New loads (kN/m <sup>2</sup> )	18	22,5	58,1	New moments	181,4	Mrd=188,3

With the original reinforcement, the beam didn't can't support the new requirements ( $Md=181.4 > 166.4=Mu$ ), so it was needed the strengthening with CFRP, and now it's perfectly correct ( $Mrd=188.3 > M'd=181.4$ )

ULS checking		SLS checking	
$Mrd = 188.33 \text{ kNm} > M'd = 181.4 \text{ kNm}$	✓	$\sigma_{c,r} = 23.26 \geq 0.6 \cdot 25(f_{ck}) = 15$	✗
$f_{yd}/E_s = 2.12 \cdot 10^{-3} \leq 14.42 \cdot 10^{-3} = \epsilon_s$	✓	$\sigma_{c,qp} = 14.21 \geq 0.45 \cdot 25(f_{ck}) = 11.25$	✗
$\epsilon_f = 4.32 \cdot 10^{-3} \leq 1.8 \cdot 10^{-2} = \epsilon_{fud}$	✓	$\sigma_s = 299.28 \leq 0.8 \cdot f_{yk} = 0.8 \cdot 500 = 400$	✓
$\xi = x/d = 0.44 < 0.45 = \xi_{(c25/30)}$	✓	$\sigma_f = 44.36 \leq 0.8 \cdot f_{fk} = 0.8 \cdot 3100 = 2480$	✓

In the ULS checking, the equations of the yield strength of the steel working under tension ( $E_s/f_{yd}$ ) and the deformation of the CFRP laminate ( $\epsilon_f$ ), less than the ultimate deformation ( $\epsilon_{s1}$  and  $\epsilon_{fud}$  respectively).  $\xi$  is to check the correct ductility of C25/30 concrete. All equations are correct.

In the SLS checking is necessary to prevent welding of the steel at service under rare load combination ( $\sigma_s$ ) and the same with the CFRP under quasi permanent load combination. These equations are valid, but the others ( $\sigma_{c,r}$  and  $\sigma_{c,qp}$ ) are incorrect and could be excessive compression, producing longitudinal cracks and irreversible strains. They depend too much of the concrete type and the dimensions. The concrete can't change it, but its possible hat increasing the dimension of the beam we could correct the equations.

## 9.2 BEAM SIMPLY SUPPORT – ANCHORED (MAX. CFRP FORCE AND LENGTH)

All data are taken from the previous example.

- $N_{fa,max}$  = maximum CFRP force which can be anchored

$$N_{fa,max} = \alpha \cdot c_1 \cdot k_c \cdot k_b \cdot b \cdot \sqrt{E_f \cdot t_f \cdot f_{ctm}}$$

$\alpha$  = Reduction factor;  $\alpha = 0.9$

$k_c$  = State of compaction of concrete;  $k_c = 1$  is good

$k_b$  = Geometry factor

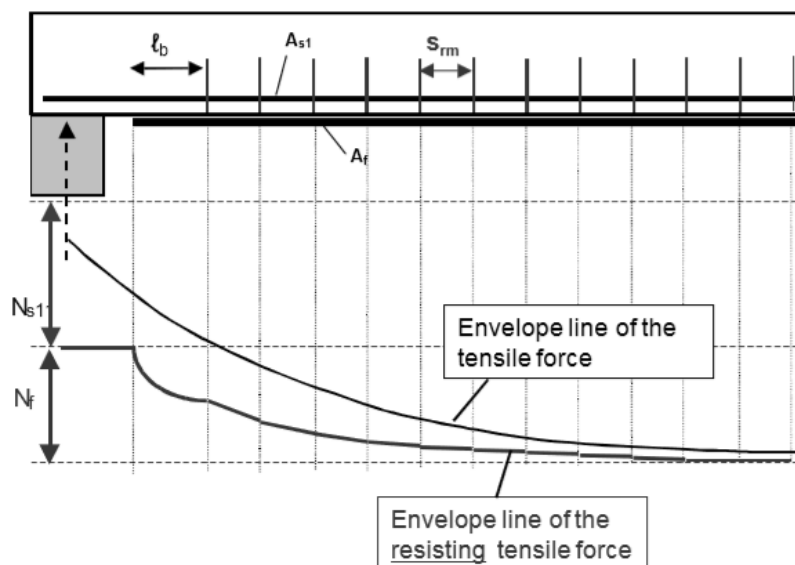
$$k_b = 1.06 \cdot \sqrt{\frac{2 - \frac{b_f}{b}}{1 + \frac{b_f}{400}}} \geq 1 \rightarrow k_b = 1.06 \cdot \sqrt{\frac{2 - \frac{50}{250}}{1 + \frac{50}{400}}} \rightarrow k_b = 1.34$$

$c_1$  and  $c_2$ : to calibrate with test results; CFRP strip  $\rightarrow c_1 = 0.64$ ;  $c_2 = 2$

$$N_{fa,max} = 0.9 \cdot 0.64 \cdot 1 \cdot 1.34 \cdot 250 \cdot \sqrt{170000 \cdot 1.4 \cdot 2.6} \rightarrow N_{fa,max} = 151789.74N$$

- $L_{b,max}$  = maximum anchorage length

$$L_{b,max} = \sqrt{\frac{E_f \cdot t_f}{c_2 \cdot f_{ctm}}} = \sqrt{\frac{170000 \cdot 1.4}{2 \cdot 2.6}} \rightarrow L_{b,max} = 213.93mm$$



### 9.3 BEAM SIMPLY SUPPORT – SHEAR STRENGTHENING

All data are taken from the first example.

#### Initial loads

$$G = 18 \text{ kN/m} \cdot 1.35 (\gamma G) = 24.3 \text{ kN/m}$$

$$Q = 15 \text{ kN/m} \cdot 1.5 (\gamma Q) = 22.5 \text{ kN/m}$$

$$q = G + Q = 24.3 + 22.5 \rightarrow q = 46.8 \text{ kN/m}$$

First, we calculate the design moment required:

$$\text{In the support} \rightarrow V_{sd} = q \cdot \frac{L}{2} = 46.8 \cdot \frac{5}{2} \rightarrow V_{sd} = 117 \text{ kN}$$

$$\text{In the distance } d \text{ (} d = 0.45 \text{ m)} \rightarrow V_{sd} = \left( q \cdot \frac{L}{2} \right) - q \cdot d = \left( 46.8 \cdot \frac{5}{2} \right) - 46.8 \cdot 0.45$$

$$\boxed{V_{sd} = 95.94 \text{ kN}}$$

Now, we have to calculate the design shear resistance of the member without shear reinforcement (only the concrete)

$$V_{cd} = [\tau_{rd} \cdot k \cdot (1.2 + 40 \cdot p_l) + 0.15 \cdot \sigma_{cp}] \cdot b_w \cdot d$$

$$\tau_{rd} = 0.3$$

$$k = 1.6 - d \geq 1$$

$$p_l = \frac{A_{sl}}{b \cdot d} \leq 0.02$$

$$\sigma_{cp} = \frac{N_{ed}}{A_c} < 0.2 \cdot f_{cd}; \text{ } N_{ed} \text{ is axial force in cross section in this case } \sigma_{cp} = 0$$

$$V_{cd} = \left[ 0.3 \cdot (1.6 - 0.45) \cdot \left( 1.2 + 40 \cdot \frac{1257}{250 \cdot 370} \right) + 0.15 \cdot 0 \right] \cdot 250 \cdot 370 = 55.64 \text{ kN}$$

We check if the contribution of concrete is enough to cover the design moment

$$V_{cd} = 55.64 \text{ kN} < 95.94 \text{ kN} = V_{sd}$$

The contribution of concrete alone is not enough to withstand the shear stress, so it is necessary to add the shear reinforcement for checking.

$$\boxed{\text{Shear reinforcement} \rightarrow \varnothing 6/150 \text{ mm} \rightarrow A_{sw} = 56 \text{ mm}^2}$$

9.2.2(5) Eurocode 1.

$$\rightarrow p_{w, \min} = \frac{0.08 \cdot \sqrt{f_{ck}}}{f_{yk}} = \frac{0.08 \cdot \sqrt{25}}{50} \rightarrow p_{w, \min} = 0.0008 \text{ mm}^2$$

$$p_w = \frac{A_{sw}}{s \cdot b_w \cdot \sin \alpha} = \frac{56}{150 \cdot 250 \cdot \sin 45} = 0.0021$$

$$\boxed{p_w = 0.0021 > 0.0008 = p_{w, \min} \text{ is OK}}$$

Now, we must calculate the contribution of the shear reinforcement ( $V_{wd}$ )

$$V_{wd} = \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} = \frac{56 \text{ mm}^2}{150 \text{ mm}} \cdot 0.9 \cdot 370 \cdot 434.78 \rightarrow V_{wd} = 54.01 \text{ kN}$$

We check again if we can cover the design moment.

$$V_{rd} = V_{cd} + V_{wd} \geq V_{sd}$$

$$V_{rd} = 55.64 + 54.01 = 109.65 \text{ kN} > 95.94 \text{ kN} = V_{sd}$$

### New loads

Now, like in the first example, we have the same changes of needs in the building. ( $Q' = Q \cdot 1.5$ )

$$G = 18 \text{ kN/m} \cdot 1.35 (\gamma G) = 24.3 \text{ kN/m}$$

$$Q' = 15 \text{ kN/m} \cdot 1.5 = 22.5 \text{ kN/m} \cdot 1.5 (\gamma Q) = 33.75 \text{ kN/m}$$

$$q' = G + Q' = 24.3 + 33.75 \rightarrow q' = 58.05 \text{ kN/m}$$

We calculate the new design moment required:

$$\text{In the support} \rightarrow V_{sd}' = q' \cdot \frac{L}{2} = 58.05 \cdot \frac{5}{2} \rightarrow V_{sd}' = 145.125 \text{ kN}$$

$$\text{In the distance } d \text{ (} d = 0.45 \text{ m)} \rightarrow V_{sd}' = \left( q' \cdot \frac{L}{2} \right) - q' \cdot d = \left( 58.05 \cdot \frac{5}{2} \right) - 58.05 \cdot 0.45$$

$$\rightarrow V_{sd}' = 119 \text{ kN}$$

We have the values of contributions of concrete and reinforcement and check if we cover the new design moment required.

$$\rightarrow V_{cd} = 55.64 \text{ kN}$$

$$\rightarrow V_{wd} = 54.01 \text{ kN}$$

$$V_{rd} = V_{cd} + V_{wd} \geq V_{sd}$$

$$V_{rd} = 55.64 + 54.01 = 109.65 \text{ kN} < 119 \text{ kN} = V_{sd}$$

The joint capacity of the concrete and the existing shear reinforcement is lower than the shear stress of the beam, so strengthening is necessary.

### Ultimate Limit State

We calculate the CFRP contribution to shear capacity ( $V_{fd}$ )

$$V_{fd} = 0.9 \cdot \varepsilon_{fd,e} \cdot E_{fu} \cdot p_f \cdot b_w \cdot d \cdot (\cot \theta + \cot \alpha) \cdot \sin \alpha$$

$$p_f = \frac{2 \cdot t_f \cdot \sin \alpha}{b_w} = \frac{2 \cdot 1.4 \cdot \sin 45}{250} \rightarrow p_f = 0.0079$$

$$\varepsilon_{fd,e} = \frac{\varepsilon_{fk,e}}{\gamma_f} \rightarrow \varepsilon_{fk,e} = k \cdot \varepsilon_{f,e}$$

$\varepsilon_{f,e} \rightarrow$  Two options:

- Fully wrapped (or properly anchored) CFRP – FRP fracture controls

$$\varepsilon_{f,e} = 0.17 \cdot \left( \frac{f_{cm}^{2/3}}{E_{fu} \cdot p_f} \right)^{0.3} \cdot \varepsilon_{fu} = 0.17 \cdot \left( \frac{33^{2/3}}{170000 \cdot 0.0079} \right)^{0.3} \cdot 0.018$$

$$\varepsilon_{f,e} = 0.0007 \rightarrow \varepsilon_{fk,e} = k \cdot \varepsilon_{f,e} = 0.8 \cdot 0.0007 = 0.000567$$

$$\varepsilon_{fd,e} = \frac{\varepsilon_{fk,e}}{\gamma_f} = \frac{0.000567}{1.3} = 0.000436$$

- Side or U – Shaped CFRP jackets

$$\varepsilon_{f,e} = \min \left[ 0.65 \cdot \left( \frac{f_{cm}^{\frac{2}{3}}}{E_{fu} \cdot p_f} \right)^{0.56} \cdot 10^{-3}; 0.17 \cdot \left( \frac{f_{cm}^{\frac{2}{3}}}{E_{fu} \cdot p_f} \right)^{0.3} \cdot \varepsilon_{fu} \right]$$

$$\min \left[ 0.65 \cdot \left( \frac{33^{2/3}}{170000 \cdot 0.0079} \right)^{0.56} \cdot 10^{-3}; 0.17 \cdot \left( \frac{33^{2/3}}{170000 \cdot 0.0079} \right)^{0.3} \cdot 0.018 \right]$$

$$\varepsilon_{f,e} = \min[0.0000424; 0.0007] \rightarrow \varepsilon_{f,e} = 0.0000424$$

$$\varepsilon_{f,e} = 0.0007 \rightarrow \varepsilon_{fk,e} = k \cdot \varepsilon_{f,e} = 0.8 \cdot 0.0000424 = 0.00003392$$

$$\varepsilon_{fd,e} = \frac{\varepsilon_{fk,e}}{\gamma_f} = \frac{0.00003392}{1.3} = 0.000026$$

We introduce the values in the equation of the CFRP contribution:

$$V_{fd} = 0.9 \cdot \varepsilon_{fd,e} \cdot E_{fu} \cdot p_f \cdot b_w \cdot d \cdot (\cot\theta + \cot\alpha) \cdot \text{sen}\alpha$$

- Fully wrapped

$$0.9 \cdot 0.000436 \cdot 170000 \cdot 0.0079 \cdot 250 \cdot 370 \cdot (\cot 45 + \cot 45) \cdot \text{sen} 45$$

$$V_{fd} = 68938.48 \text{ N} = 68.93 \text{ kN}$$

Now, we sum all the contributions and check if the requested moment is overcome

$$V_{rd} = V_{cd} + V_{wd} + V_{fd} = 55.64 + 54.01 + 68.93 = 178.58 \text{ kN}$$

$$V_{rd} = 178.58 \text{ kN} > 119 \text{ kN} = V_{sd} \text{ is OK}$$

- Side or U – Shaped

$$0.9 \cdot 0.000026 \cdot 170000 \cdot 0.0079 \cdot 250 \cdot 370 \cdot (\cot 45 + \cot 45) \cdot \text{sen} 45$$

$$V_{fd} = 411.01 \text{ N} = 4.11 \text{ kN}$$

Now, we sum all the contributions and check if the requested moment is overcome

$$V_{rd} = V_{cd} + V_{wd} + V_{fd} = 55.64 + 54.01 + 4.11 = 113.76 \text{ kN}$$

$$V_{rd} = 113.76 \text{ kN} < 119 \text{ kN} = V_{sd} \text{ is not OK}$$

### Serviceability Limit State

The externally bonded reinforcement should not debond at the serviceability limit state. This is important, so that problems related to moisture penetration, crack propagation, noise from debonding etc. may be avoided. To verify this, the strain in the CFRP in the serviceability limit state,  $\varepsilon_{fk,e}$ , should be limited to  $\frac{0.8 \cdot f_{yk}}{E_s}$ , unless otherwise provided and verified by the CFRP system supplier.

$$\varepsilon_{fk,e} \leq \frac{0.8 \cdot f_{yk}}{E_s}$$

- Fully wrapped

$$0.000567 \leq \frac{0.8 \cdot 500}{205000} \rightarrow 0.000567 \leq 0.0019 \text{ is OK}$$

- Side or U – Shaped

$$0.00003392 \leq \frac{0.8 \cdot 500}{205000} \rightarrow 0.00003392 \leq 0.0019 \text{ is OK}$$

### Conclusions



	G	Q	q		Contribution required	V <sub>cd</sub>	55,64
Origin loads (kN/m <sup>2</sup> )	18	15	46,8	V <sub>d</sub> (kN)	95,94	V <sub>wd</sub>	50,01
New loads (kN/m <sup>2</sup> )	18	22,5	58,1	V <sub>d'</sub> (kN)	119	V <sub>fd</sub> (Full wrapped)	68,93
						V <sub>fd</sub> (U)	4,11

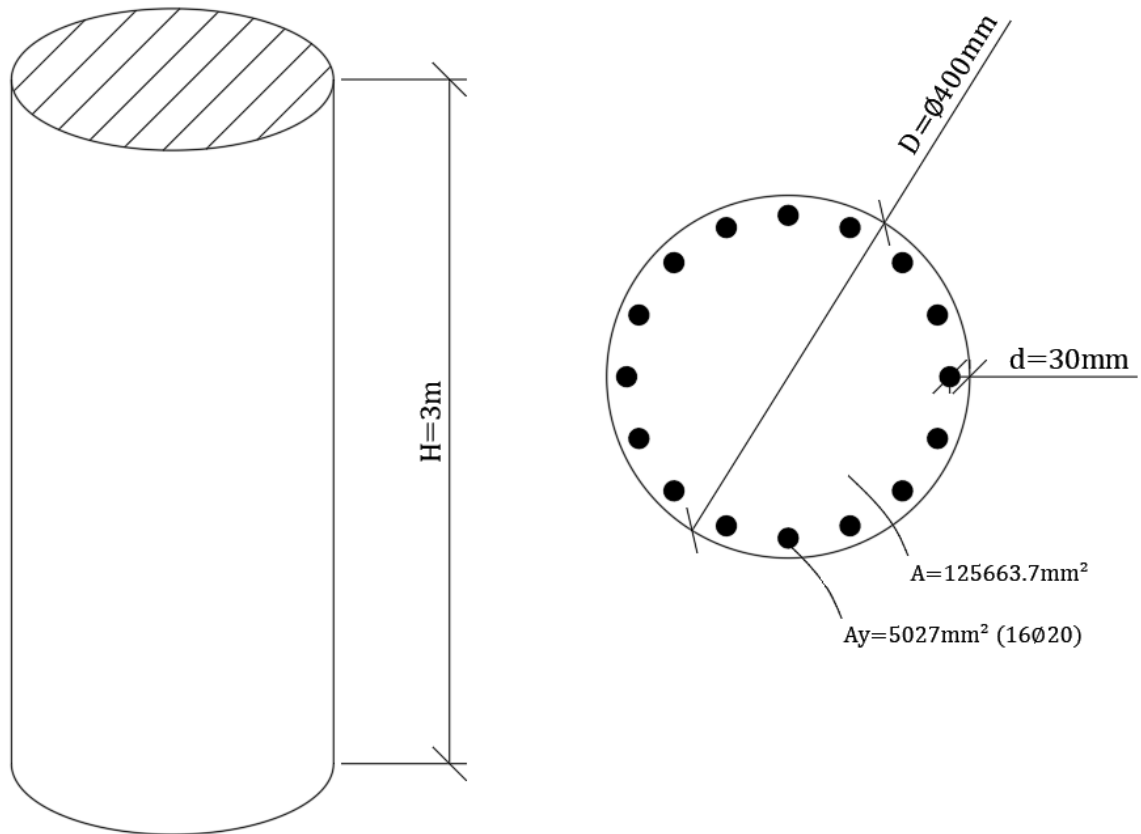
With the original reinforcement, the beam didn't can't support the new requirements ( $V_{cd} + V_{wd} = 109.65 < 119 = V_{d'}$ ), so it was needed the strengthening with CFRP.

ULS checking			
Full Wrapped		Side or U-Shaped	
$V_{rd} = 178.58 \text{ kN} > 119 \text{ kN} = V_{d'}$	✓	$V_{rd} = 113 \text{ kN} < 119 \text{ kN} = V_{d'}$	✗
SLS checking			
Full Wrapped		Side or U – Shaped	
$0.000567 = \epsilon_{fk, e} \leq 0.8 \cdot f_{yk}/E_s = 0.0019$	✓	$0.0000392 = \epsilon_{fk, e} \leq 0.8 \cdot f_{yk}/E_s = 0.0019$	✓

Using the full wrapped the design shear resistance is guaranteed ( $V_{cd} + V_{wd} + V_{fd} = 178.58 > 119 = V_{d'}$ ) but only using CFRP in side or U-shaped is not enough to cover the contribution required ( $V_{cd} + V_{wd} + V_{fd} = 113 < 119 = V_{d'}$ ), so only we can put the full wrapped.

In both cases the strain in the CFRP in the serviceability limit state ( $\epsilon_{fk, e}$ ) is correctly limited to to  $\frac{0.8 \cdot f_{yk}}{E_s}$ , unless otherwise provided and verified by the FRP system supplier.

#### 9.4 CIRCULAR COLUMN – CONFINED CAPACITY COMPARISON.



##### Materials

Concrete: C25/30

$f_{ck} = 25\text{MPa}$ ;  $f_{cd}=f_{ck}/\gamma_c = 25/1.5 = 16.67\text{MPa}$ ;  $f_{co}=14.11\text{MPa}$   $E_c=31\text{GPa}$

Steel: feB44K

$f_{yk} = 437\text{MPa}$ ;  $f_{yd}=f_{yk}/\gamma_y = 500/1.15 = 374\text{MPa}$ ;  $E_s=205\text{GPa}$

##### Dimensions and reinforcement

$H=3.00\text{ m}$ ;  $D=0.4\text{m}$ ;  $A=125663.7\text{mm}^2$ ;

$A_y=5027\text{mm}^2$  (16Ø20);  $d=0.03\text{m}$ ;

##### CFRP: Mapei C UNI-AX 600

$F_u=4830\text{ MPa}$ ;  $E_f=230000\text{ Mpa}$ ;  $t_f=0.333\text{mm}$

$\epsilon_{frp,u}=0.021$ ;  $\epsilon_{fd,rid(R)}=0.004$ ;  $\epsilon_{fd,rid(D)}=0.013$ ;

$\eta=0.95$ ;  $\gamma_f=1.1$ ;  $\gamma_{r,d}=1.1 \rightarrow$  Continuous wrapped

$$p_f = \frac{4 \cdot t_f}{d_f} = \frac{4 \cdot 0.333}{400} \rightarrow p_f = 0.00333$$

$$k_h = 1; \quad k_v = 1; \quad k_\alpha = 1; \quad \rightarrow k_{eff} = k_h \cdot k_v \cdot k_\alpha; \quad \rightarrow k_{eff} = 1$$

Coefficient for high eccentricity

$$f_1 = p_f \cdot 3E_f \cdot \epsilon_{rd,rid} \cdot 0.5 = 0.00333 \cdot 230000 \cdot 0.013 \cdot 0.5 \rightarrow f_1 = 4.97$$

$$f_{1,eff} = f_1 \cdot k_{eff} = 4.97 \cdot 1 \rightarrow f_{1,eff} = 4.97$$

$$e_{ccu} = \left( 0.0035 + 0.015 \cdot \sqrt{\frac{f_{1, \text{eff}}}{f_{co}}} \right) \cdot 1000 = \left( 0.0035 + 0.015 \cdot \sqrt{\frac{4.97}{14.11}} \right) \cdot 1000$$

$e_{ccu} = 12.4$  per thousand

Coefficient for low eccentricity

$$f_{ccd} = f_{co} \cdot \left[ 1 + 2.6 \cdot \left( \frac{f_{1, \text{eff}}}{f_{co}} \right)^{2/3} \right] = 14.11 \cdot \left[ 1 + 2.6 \cdot \left( \frac{1.5318}{14.11} \right)^{2/3} \right] \rightarrow f_{ccd} = 22.4$$

$$\frac{f_{1, \text{eff}}}{f_{ccd}} = \frac{1.5318}{22.4} = 0.067$$

Axial force taken over by the confined section

$$NR_{cc, d} = \frac{\left( A \cdot \frac{f_{ccd}}{\gamma_{R, d}} \right) + (A_y \cdot f_{yd})}{1000} = \frac{\left( 125663.7 \cdot \frac{22.4}{1.1} \right) + (5027 \cdot 374)}{1000} = 4439 \text{ kN}$$

The last axial force taken over by the unfinished section

$$NR_{c, d} = \frac{(A \cdot f_{co}) + (A_y \cdot f_{yd})}{1000} = \frac{(125663.7 \cdot 14.11) + (5027 \cdot 374)}{1000} = 3653.2 \text{ kN}$$

The axial force input assumed by confinement (consolidation)

$$N_{frp} = NR_{cc, d} - NR_{c, d} = 4439 \text{ kN} - 3653.2 \text{ kN} \rightarrow N_{frp} = 785 \text{ kN}$$

Comparison with Excel of Mapei

Date de intrare		
Caracteristici geometrice		
Tipul sectiunii	Circulara	▼
Diametrul (D)	400	mm
Aria sectiunii de beton	125600	mm <sup>2</sup>
Acoperirea cu beton (d)	30	mm
Aria totala de armatura A <sub>s</sub>	5027	mm <sup>2</sup>

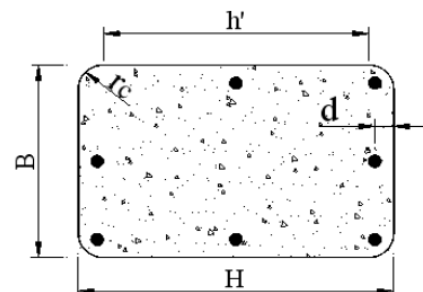


ADESIVI SIGILLANTI PRODOTTI CHIMICI PER L'EDILIZIA

**Asistenta Tehnica - Departament Produse Speciale**  
 Client: SC APOLODOR S.A  
 Intocmit: Ing. Cristi Cartas- MAPEI ROMANIA  
 Santier: APOLODOR

Materiale		Eforturi de calcul MPa	
Beton clasa	C 25/30	14,11	1,5
Otel	FeB 44 k	374	1,15

Tipul compozitului	Tipul fibrei	Expunere
MapeWrap C UNI-AX 600	Carbon	Interna
Tip de aplicare		Dispunere consolidare
Sistem de consolidare agrementat		Continua



CARACTERISTICI FRP/FRG	
$\sigma$ la rupere	4830 MPa
Modul de elasticitate	230000 MPa
grosime ( $t_f$ )	0,333 mm
$\varepsilon$ la rupere, $\varepsilon_{frp,u}$	0,021
$\varepsilon$ de calcul, $\varepsilon_{fd,rid}$ : (Resistenta)	0,004
$\varepsilon$ de calcul, $\varepsilon_{fd,rid}$ : (Ductilitate)	0,013
Coefficient partial de siguranta FRP ( $\gamma_f$ )	1,1
Factor ce tine cont de expunere ( $\eta_a$ )	0,95
Coefficient partial ( $\gamma_{R,d}$ )	1,1

$\rho_f$	0,00333
$k_H$	1,00
$k_V$	1,00
$k_c$	1,00
$k_{eff}$	1,00
Coeficienti pentru excentricitate mare	
$f_1$	4,83
$f_{1,eff}$	4,825
$\varepsilon_{ccu}$	<b>12,27 ‰</b>
Coeficienti pentru excentricitate mica	
$f_1$	1,53
$f_{1,eff}$	1,532
$f_{ccd}$	<b>22,46 MPa</b>
$f_{1,eff}/f_{cd}$	0,109

$N_{Rcc,d}$	<b>4443,9 kN</b>	Forta axiala preluata de sectiunea confinata
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$N_{Rc,d}$	<b>3651,7 kN</b>	Forta axiala ultima preluata de sectiunea neconfinata
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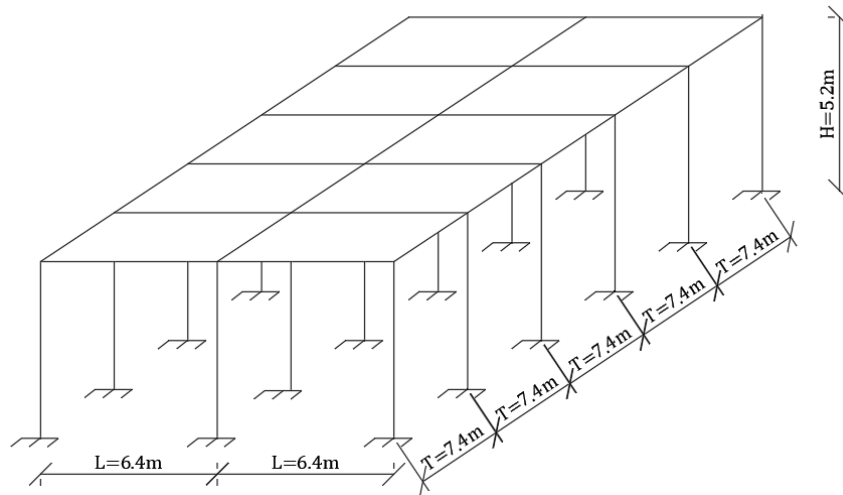
$N_{FRP}$	<b>792,2 kN</b>	Aportul de forta axiala preluata de confinare (consolidare)
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### Conclusions

	Mapei (kN)	Written (kN)
Axial force confined section	4443,9	4439
Axial force unconfined section	3651,7	3653,2
Axial force CFRP	792,2	785

The results are very similar, that's mean that a cylindrical column with diameter 400mm and reinforced with 16 $\phi$ 20 can be support new load that transmit an overload axial force until 785kN to de column.

## 9.5 SLAB ONE DIRECTION – FLEXURAL STRENGTHENING.



### Design information

Dimensions

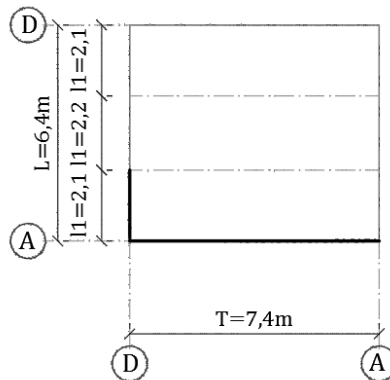
Length =  $L=6.4\text{m}$

Thickness =  $T=7.4\text{m}$

Height =  $H=5.2\text{m}$

Minimum fire resistance = 60 minutes (R60);

$$l = \frac{L}{n} \rightarrow L = 6.4\text{m} ; n = 3 \rightarrow l = \frac{6.4\text{m}}{3} ; l = 2.13\text{m}$$



Verification ( $l_1 = 2.1\text{m}$ ):  $\frac{T}{l_1} = \frac{7.4\text{m}}{2.1\text{m}} = 3.52 > 2$  One way slab

Verification ( $l_2 = 2.2\text{m}$ ):  $\frac{T}{l_2} = \frac{7.4\text{m}}{2.2\text{m}} = 3.36 > 2$  One way slab

### Materials

Concrete C 25/30  $\rightarrow f_{ck} = 25 \text{ N/mm}^2$  ;  $\gamma_c = 1.5$

$$f_{cd} = \frac{25 \text{ N/mm}^2}{1.5} ; f_{cd} = 16.67 \text{ N/mm}^2$$

Steel S500  $\rightarrow f_{yk} = 500 \text{ N/mm}^2$  ;  $\gamma_s = 1.15$

$$f_{yd} = \frac{500 \text{ N/mm}^2}{1.15} ; f_{cd} = 434.783 \text{ N/mm}^2$$

### Predimensioning of the concrete sections

- The thickness of the slab

The rigidity condition  $hf \geq \frac{2.1m}{35}$  ;  $hf \geq 0.06m$  ;  $hf \geq 60mm$

The technological condition  $hf \geq 50mm$

Fire resistance condition (R60)  $hf \geq 80mm$

- The dimensions of the transversal section of the secondary beam

The rigidity condition  $hn \geq \frac{7.4m}{20}$  ;  $hn \geq 0.37m$  ;  $hn \geq 400mm$

The technological condition  $hn$  – multiple of 50mm;  $bn \geq 200mm$

Fire resistance condition (R60)  $b \geq 120mm$

Is chosen  $hn=400mm$  and  $bn=200mm$ . The proportion  $h/b$  is 2. The section is in T

### Load evaluation in slab

- Permanent loads – dead loads (qk)

Self-weight of the slab

$$hf \cdot 1m \cdot 1m \cdot \gamma_{ck} = 0.08m \cdot 1m \cdot 1m \cdot \frac{25kN}{m^3} = 2 \frac{kN}{m^2}$$

The weight of the floor:  $0.94 \frac{kN}{m^2}$

Total permanent loads:  $2.94 \frac{kN}{m^2}$

- Variable loads (gk)

Total variable loads:  $5.2 \frac{kN}{m^2}$

Design values

$$Pd = \gamma_q \cdot qk + \gamma_g \cdot gk ; \quad \gamma_q = 1.5 ; \quad \gamma_g = 1.35$$

$$Pd = 1.35 \cdot 2.94 \frac{kN}{m^2} + 1.5 \cdot 5.2 \frac{kN}{m^2} \rightarrow Pd = 11.769 \frac{kN}{m^2}$$

### Static design

Openings

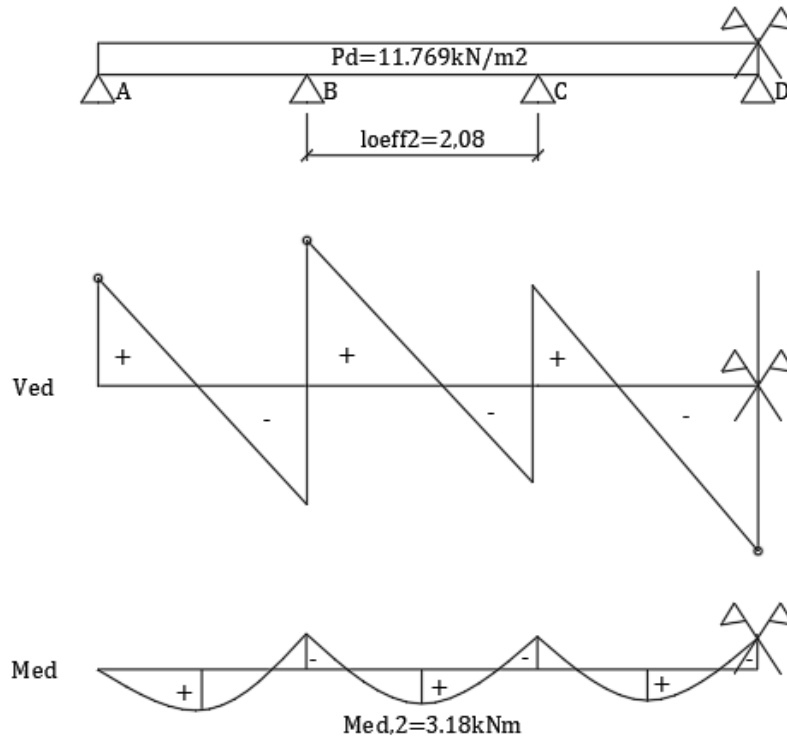
$$ln2 = l2 - 2 \cdot \frac{bn}{2} = 2200mm - 2 \cdot \frac{200mm}{2} = 2000mm$$

Calculate the design span

$$loeff2 = ln2 + 2 \cdot \frac{hf}{2} = 2000mm + 2 \cdot \frac{80mm}{2} = 2080mm$$

Bending moment

$$Med,2 = \frac{1}{16} \cdot Pd \cdot loeff2^2 \cdot 1m = \frac{1}{16} \cdot 11.769 \frac{kN}{m^2} \cdot 2.08^2m \cdot 1m = 3.18kNm$$



Dimensioning of resistance reinforcement

Dimensioning bending moment

$$d = hf - C_{nom} - \frac{\varnothing_{sl}}{2}$$

$$C_{nom} = C_{min} + \Delta_{ctol} \rightarrow C_{min} = 10\text{mm}; \Delta_{ctol} = 10\text{mm}$$

$$C_{nom} = 10\text{mm} + 10\text{mm} = 20\text{mm}$$

$$\varnothing_{sl} \min\{0.1 \cdot hf + 2\text{mm} = 10\text{mm}; 6\text{mm}\} = 6\text{mm}$$

$$d = 80\text{mm} - 20\text{mm} - \frac{6\text{mm}}{2}; \quad d = 57\text{mm}$$

Minimum distance till the centroid

Fire: R60, one direction  $\rightarrow a, \min = 20\text{mm}$

$$a, \text{eff} = 20\text{mm} + \frac{\varnothing_{sl}}{2} = 20 + \frac{6}{2} \rightarrow a, \text{eff} = 23\text{mm} \rightarrow a, \text{eff} > a, \min; \quad b = 1\text{m}$$

The slab will be reinforced using meshes:

SPAN 2 (Med,2=3.18kNm)

$$\mu = \frac{Med,2}{b \cdot d^2 \cdot f_{cd}} = \frac{3.18 \cdot 10^6}{1000\text{mm} \cdot 57^2\text{mm} \cdot 16.67 \frac{\text{N}}{\text{mm}^2}} \rightarrow \mu = 0.058 < 0.372 (\mu \text{ lim S500})$$

$$w = 1 - \sqrt{1 - 2 \cdot \mu} = 1 - \sqrt{1 - 2 \cdot 0.058} \rightarrow w = 0.06$$

$$A_{sl} = w \cdot b \cdot d \cdot \frac{f_{cd}}{f_{yd}} = 0.06 \cdot 1000 \cdot 57 \cdot \frac{16.67}{434.78} \rightarrow A_{sl} = 132.32\text{mm}^2 = 1.32\text{cm}^2$$

$\varnothing 6/18$  ( $A_{sl, \text{eff}} = 1.57\text{cm}^2$ )

### New load evaluation in slab

Permanents loads are the same.

New variable loads =  $g_k + 30\% \cdot g_k$ .

Total permanent loads:  $2.94 \frac{\text{kN}}{\text{m}^2}$

Total variable loads:  $5.2 \frac{\text{kN}}{\text{m}^2} \cdot 1.3 = 6.76 \frac{\text{kN}}{\text{m}^2}$

New design values

$Pd' = \gamma_q \cdot q_k + \gamma_g \cdot g_k'$ ;  $\gamma_q = 1.5$ ;  $\gamma_g = 1.35$

$$Pd' = 1.35 \cdot 2.94 \frac{\text{kN}}{\text{m}^2} + 1.5 \cdot 6.76 \frac{\text{kN}}{\text{m}^2} \rightarrow Pd' = 14.109 \frac{\text{kN}}{\text{m}^2}$$

### New static design

Bending moment

$$Med', 2 = \frac{1}{16} \cdot Pd \cdot loeff2^2 \cdot 1m = \frac{1}{16} \cdot 14.109 \frac{\text{kN}}{\text{m}^2} \cdot 2.08^2 m \cdot 1m = 3.81 \text{kNm}$$

New reinforcement requirements

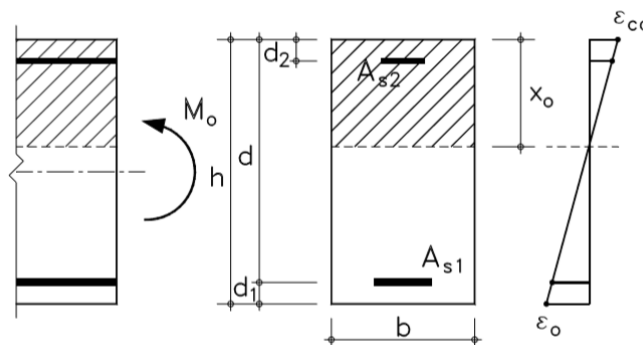
$$\mu' = \frac{Med', 2}{b \cdot d^2 \cdot fcd} = \frac{3.81 \cdot 10^6}{1000 \text{mm} \cdot 57^2 \text{mm} \cdot 16.67 \frac{\text{N}}{\text{mm}^2}} \rightarrow \mu' = 0.073 < 0.372 (\mu \text{ lim S500})$$

$$w' = 1 - \sqrt{1 - 2 \cdot \mu'} = 1 - \sqrt{1 - 2 \cdot 0.073} \rightarrow w' = 0.073$$

$$Asl' = w' \cdot b \cdot d \cdot \frac{fcd}{fyd} = 0.073 \cdot 1000 \cdot 57 \cdot \frac{16.67}{434.78} \rightarrow Asl' = 159.56 \text{mm}^2 = 1.59 \text{cm}^2$$

$\emptyset 6/18 \rightarrow Asl, \text{eff} = 1.57 \text{cm}^2 < Asl' = 1.59 \text{cm}^2$  is not OK

The slab does not support the new moment of solicitation since the last moment is less than the moment that it must resist as a result of applying the overload. The slab has to be reinforced and the application of the reinforcement by means of carbon fiber laminates (CFRP) applied on the underside of the slab is opted for.



Neutral axis depth =  $X_o$

$$\frac{1}{2} \cdot b \cdot X_o^2 + (\alpha_s - 1) \cdot A_{s2} \cdot (X_o - d_2) = \alpha_s \cdot A_{s1} \cdot (d - X_o)$$



$$\alpha_s = \frac{E_s}{E_c} = \frac{205}{31} = 6.61$$

$$M_o = \frac{1}{16} \cdot q \cdot l_{\text{eff}}^2 \cdot 1\text{m} = \frac{1}{16} \cdot 2.94 \frac{\text{kN}}{\text{m}^2} \cdot 2.08^2 \text{m} \cdot 1\text{m} \rightarrow M_o = 0.7949 \text{kNm}$$

(No load safety factors are applied)

$$\frac{1}{2} \cdot 1000 \cdot X_o^2 + (6.61 - 1) \cdot 0 \cdot (X_o - 23) = 6.61 \cdot 157 \cdot (57 - X_o)$$

$$500 \cdot X_o^2 = 59152.89 - 1037.77 \cdot X_o$$

$$500 \cdot X_o^2 + 1037.77 \cdot X_o - 59152.89 = 0$$

$$X_o = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \rightarrow \boxed{X_o = 9.88 \text{ mm}}$$

Next, we calculate the moment of inertia of the cracked section that will be necessary to calculate the deformation of the concrete  $\varepsilon_o$ :

$I_{o2}$  = Moment of inertia of the transformed cracked section

$$I_{o2} = \frac{b \cdot X_o^3}{3} + (\alpha_s - 1) \cdot A_{s2} \cdot (X_o - d_2)^2 + \alpha_s \cdot A_{s1} \cdot (d - X_o)^2$$

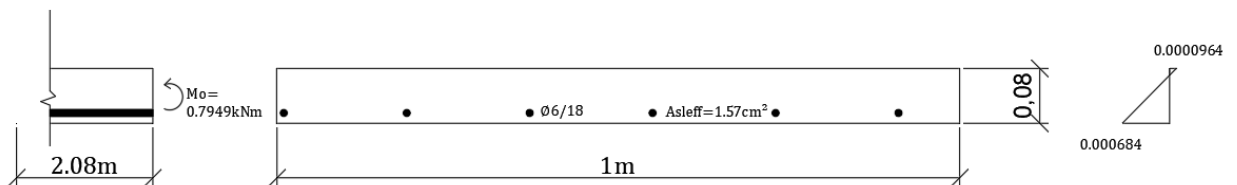
$$I_{o2} = \frac{100 \cdot 9.88^3}{3} + 6.61 \cdot 157 \cdot (57 - 9.88)^2$$

$$I_{o2} = 1321476.75 + 2304154.91 \rightarrow I_{o2} = 2625631.66 \text{ mm}^4$$

$$\varepsilon_{co} = \frac{M_o \cdot X_o}{E_c \cdot I_{o2}} = \frac{794900 \cdot 9.88}{31000 \cdot 2625631.66} \rightarrow \boxed{\varepsilon_{co} = 9.64 \cdot 10^{-5}}$$

Finally, by deformation compatibility, we calculate the deformation of the concrete in the extreme fiber to tension:

$$\varepsilon_o = \varepsilon_{co} \cdot \frac{h - X_o}{X_o} = 9.64 \cdot 10^{-5} \cdot \frac{80 - 9.88}{9.88} \rightarrow \boxed{\varepsilon_o = 6.84 \cdot 10^{-4}}$$



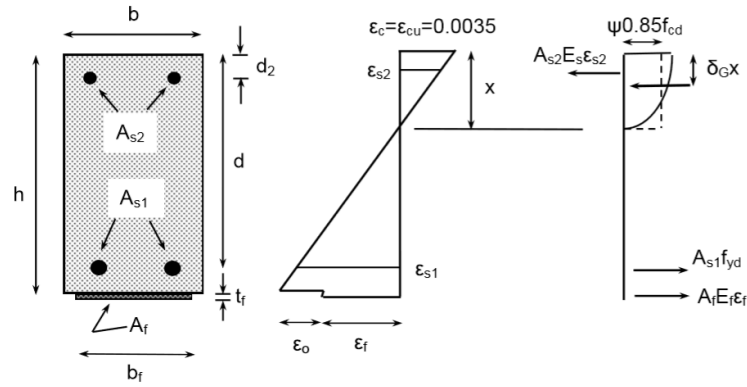
### Analysis of Ultimate Limit State

According to FIB, the desirable working principle is to reach the elastic limit of the steel followed by the crushing of the concrete and thus we will perform the calculation. We start with the moment after the overload, with the increase of variable loads.

CFRP that we use: CARBOPLATE E170

Thickness:  $t_f = 1.4 \text{ mm}$ ; Width:  $b_f = 50 \text{ mm}$ ; Area:  $A_f = 70 \text{ mm}^2$

$\sigma_{fu} = f_{fu} = 3100 \text{ Mpa} = 3.1 \text{ Kn/mm}^2$ ;  $\varepsilon_{fu} = 0.018$ ;  $E_{fu} = 170 \text{ GPa} = 170 \text{ kN/mm}^2$



$$0.85 \cdot \varphi \cdot f_{cd} \cdot b \cdot x + A_{s2} \cdot E_s \cdot \epsilon_{s2} = A_{s1} \cdot f_{yd} + A_f \cdot E_f \cdot \epsilon_f$$

$$\varphi = 0.8$$

To solve the depth of the neutral axis, we will start from an estimated value that we will introduce first in the equations to find the deformations in the compression steel and in the laminate. Calculated these values, we will introduce them into the equation of the neutral axis to find the value of it, which must coincide with the previous one. It is a process of iteration until reaching the value of x that satisfies the principle in the ultimate limit state of deformation compatibility and internal balance of forces (first we start from 30mm).

$$\epsilon_f = \epsilon_{cu} \cdot \frac{h - x}{x} - \epsilon_o = 0.0035 \cdot \frac{80 - 30}{30} - 6.84 \cdot 10^{-4} \rightarrow \epsilon_f = 0.0057641$$

We introduce the values in the equation of the neutral axis:

$$0.85 \cdot \varphi \cdot f_{cd} \cdot b \cdot x + A_{s2} \cdot E_s \cdot \epsilon_{s2} = A_{s1} \cdot f_{yd} + A_f \cdot E_f \cdot \epsilon_f$$

$$0.85 \cdot 0.8 \cdot 16.67 \cdot 1000 \cdot x = 157 \cdot 434.78 + 70 \cdot 170000 \cdot 0.0057641$$

$$11335.6 \cdot x = 68260 + 68592.79$$

$$\rightarrow x = 12.07 \neq 30$$

The first depth proposal of the neutral axis is not correct. Let's estimate as second proposal, x = 12.07 mm:

$$\epsilon_f = \epsilon_{cu} \cdot \frac{h - x}{x} - \epsilon_o = 0.0035 \cdot \frac{80 - 12.07}{12.07} - 6.84 \cdot 10^{-4} \rightarrow \epsilon_f = 0.019$$

We introduce the new values in the equation of the neutral axis:

$$0.85 \cdot \varphi \cdot f_{cd} \cdot b \cdot x + A_{s2} \cdot E_s \cdot \epsilon_{s2} = A_{s1} \cdot f_{yd} + A_f \cdot E_f \cdot \epsilon_f$$

$$0.85 \cdot 0.8 \cdot 16.67 \cdot 1000 \cdot x = 157 \cdot 434.78 + 70 \cdot 170000 \cdot 0.019$$

$$11335.6 \cdot x = 68260 + 226100$$

$$x = 25.96 \neq 12.07$$

We continue with the process until it matches the previous value:

x	25,96	12,93	24,32	13,69	23,06	14,35	22,07	14,92	21,29	15,41	20,67	15,82	20,17	16,17	19,77	16,47	19,44	16,72	19,18	16,93
ef		0,007	0,017	0,007	0,016	0,008	0,015	0,009	0,015	0,009	0,014	0,009	0,014	0,01	0,013	0,01	0,013	0,01	0,013	0,01
x		16,93	18,96	17,1	18,79	17,25	18,64	17,37	18,52	17,47	18,42	17,55	18,34	17,62	18,28	17,68	18,22	17,73	18,18	17,77
ef		0,01	0,012	0,011	0,012	0,011	0,012	0,011	0,012	0,011	0,012	0,011	0,012	0,011	0,012	0,011	0,012	0,011	0,012	0,011
x		17,77	18,14	17,8	18,11	17,83	18,08	17,85	18,06	17,87	18,04	17,89	18,03	17,9	18,02	17,91	18,01	17,92	18	17,93
ef		0,011	0,012	0,011	0,012	0,011	0,012	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011
x		17,93	17,99	17,93	17,99	17,94	17,98	17,94	17,98	17,95	17,98	17,95	17,97	17,95	17,97	17,95	17,97	17,95	17,97	17,96
ef		0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011	0,011

We already have the depth of the neutral axis in ELU, where the compression steel works in a plastic regime and the deformation of the laminate is lower than the limit deformation of the steel (10 ‰). We calculate the design resistant moment:

$$M_{rd} = A_{s1} \cdot f_{yd} \cdot (d - \delta g \cdot x) + A_f \cdot E_f \cdot \epsilon_f (h - \delta g \cdot x) + A_{s2} \cdot E_s \cdot \epsilon_{s2} \cdot (\delta g \cdot x - d_2)$$

$$\delta g = 0.4$$

$$M_{rd} = 127 \cdot 434.78 \cdot (57 - 0.4 \cdot 17.96) + 70 \cdot 170000 \cdot 0.0114 \cdot (80 - 0.4 \cdot 17.96)$$

$$M_{rd} = 2750693 + 9878218.56 \rightarrow M_{rd} = 12628911.56 \text{ Nmm}$$

$$M_{rd} = 12.62 \text{ kNm} > M'd = 3.81 \text{ kNm is OK}$$

The resistant moment of the section of the slab with the CFRP laminated reinforcement is greater than the stress to which it is loaded. The 50 mm carbon fiber reinforcement. is correct and will be placed on the axis of the lower face of the slab. As indicated in bulletin 14, for simply supported beam, the laminate will be placed over the entire length of the beam, until to a distance of the support no greater than 50 mm.

For the entire procedure to be valid, we check the following equations of the yield strength of the steel working under tension and the deformation of the CFRP laminate, which must be less than the ultimate deformation.

CFRP in the axis of the lower face (side)

$$\epsilon_{s1} = \epsilon_{cu} \cdot \frac{d - x}{x} \geq \frac{f_{yd}}{E_s} \rightarrow \epsilon_{s1} = 0.0035 \cdot \frac{57 - 17.96}{17.96} \geq \frac{434.78}{205000} \rightarrow$$

$$\rightarrow \epsilon_{s1} = 7.6 \cdot 10^{-3} \geq 2.12 \cdot 10^{-3} \text{ is OK}$$

$$\epsilon_f = \epsilon_{cu} \cdot \frac{h - x}{x} - \epsilon_0 \leq \epsilon_{fud} \rightarrow$$

$$\epsilon_f = 0.0035 \cdot \frac{80 - 17.96}{17.96} - 6.84 \cdot 10^{-4} \leq 0.018 \rightarrow$$

$$\rightarrow \epsilon_f = 1.14 \cdot 10^{-2} \leq 1.8 \cdot 10^{-2} \text{ is OK}$$

The last section of this step corresponds to verify that sufficient ductility is obtained. According to the FIB ductility criterion, we must verify that:

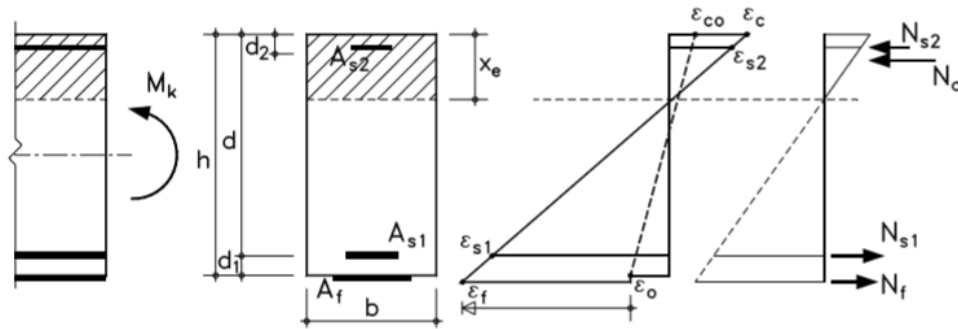
$$\xi \leq 0.45 \text{ Concrete c25/30}$$

$$\xi = \frac{x}{d} = \frac{17.96}{57} \rightarrow$$

$$0.31 < 0.45 \text{ is OK}$$

### Analysis Serviceability Limit State

Calculations to verify the serviceability limit state may be performed according to a linear elastic analysis. Reference will be made to both uncracked (state 1) and cracked sections (state 2). Whereas the neutral axis depth of RC members, according to a linear elastic calculation, is independent from the acting moment, this is no longer the case for a strengthened section as a result of the initial strains before strengthening. Assuming linear elastic material behaviour and that the concrete does not sustain tension, the cracked section analysis can be based on the next figure:



We start with the depth of the neutral axis, by means of balance of forces and compatibility of deformations:

$$\frac{1}{2} \cdot b \cdot X_e^2 + (\alpha_s - 1) \cdot A_{s2} \cdot (x_e - d_2) = \alpha_s \cdot A_{s1} \cdot (d - X_e) + \alpha_f \cdot A_f \cdot \left[ h - \left( 1 + \frac{\varepsilon_0}{\varepsilon_c} \right) + X_e \right]$$

$$\alpha_f = \frac{E_f}{E_c} = \frac{170000}{31000} = 5.48$$

$$\alpha_s = \frac{E_s}{E_c} = \frac{205000}{31000} = 6.61$$

$$\left( 1 + \frac{\varepsilon_0}{\varepsilon_c} \right) = 1$$

$$\frac{1}{2} \cdot b \cdot X_e^2 + (\alpha_s - 1) \cdot A_{s2} \cdot (x_e - d_2) = \alpha_s \cdot A_{s1} \cdot (d - X_e) + \alpha_f \cdot A_f \cdot \left[ h - \left( 1 + \frac{\varepsilon_0}{\varepsilon_c} \right) + X_e \right]$$

$$\frac{1}{2} \cdot 1000 \cdot X_e^2 = 6.61 \cdot 157 \cdot (57 - X_e) + 5.48 \cdot 70 \cdot [80 - 1 \cdot X_e]$$

$$500 \cdot X_e^2 + 1421.37 \cdot X_e - 89840.89 = 0$$

$$X_e = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \rightarrow X_e = 12.05 \text{ mm}$$

We proceed to calculate the combinations of actions in ELS. For the calculation, we take the acting loads without safety coefficient:

Different loads combinations:

- Under the rare load combination

$$\sum Q_{k,j} + P + Q_{k,1} + \sum_{i>1} \varphi_{0,i} Q_{k,i}$$

$$\sum Q_{k,j} = \text{characteristic value of permanent load}$$

P = Representative of prestressed load

Q<sub>k,1</sub> = Characteristic value of variable load 1

$\sum_{i>1} \varphi_{0,i} Q_{k,i}$  = Characteristic value of variable load, reduced.

$$\sum Q_{k,j} + Q_{k,1} = 2.94 + 6.76 = 9.7 \frac{\text{kN}}{\text{m}}$$

- Under the quasi-permanent load combination

$$\sum Q_{k,j} + P + \sum_{i>1} \varphi_{2,i} Q_{k,i}$$

$$\text{Our situation only } \sum Q_{k,j} \text{ and } \sum_{i>1} \varphi_{2,i} Q_{k,i}$$

(reduced by coefficient  $\varphi_2 = 0.3$ , category A, Table A 1.1 Eurocode 1)

$$\sum Q_{k,j} + \sum_{i>1} \varphi_{2,i} Q_{k,i} = 2.94 + (6.76 \cdot 0.3) = 4.97 \text{ kN/m}$$

Calculated the neutral axis, we have two values of  $E_c \cdot \varepsilon_c$  depending on the combination of actions. We check in this way if it meets the SLS requirement:

- Under rare load combinations

$$M_{k,r} = \frac{1}{16} \cdot q \cdot \text{coeff}^2 \cdot 1 \text{ m} = \frac{1}{16} \cdot 9.7 \frac{\text{kN}}{\text{m}^2} \cdot 2.08^2 \text{ m} \cdot 1 \text{ m} \rightarrow M_{k,r} = 2.62 \text{ kNm}$$

$$E_c \cdot \varepsilon_c = \frac{M_k}{\frac{1}{2} \cdot b \cdot X_e \cdot (1.05 \cdot d - \frac{X_e}{3})} = \frac{2620000}{\frac{1}{2} \cdot 1000 \cdot 12.05 \cdot (1.05 \cdot 57 - \frac{12.05}{3})}$$

$$E_c \cdot \varepsilon_c = \frac{2620000}{336395.83} \rightarrow E_c \cdot \varepsilon_c = 7.97 \text{ N/mm}^2$$

Stress limitation

$$\sigma_{c,r} = E_c \cdot \varepsilon_c \leq 0.6 \cdot f_{ck} \rightarrow \sigma_{c,r} = 7.97 \leq 0.6 \cdot 25 = 15 \text{ is OK}$$

- Under quasi-permanent load combinations

$$M_{k,qp} = \frac{1}{16} \cdot q \cdot \text{coeff}^2 \cdot 1 \text{ m} = \frac{1}{16} \cdot 4.97 \frac{\text{kN}}{\text{m}^2} \cdot 2.08^2 \text{ m} \cdot 1 \text{ m} \rightarrow M_{k,r} = 1.343 \text{ kNm}$$

$$E_c \cdot \varepsilon_c = \frac{M_k}{\frac{1}{2} \cdot b \cdot X_e \cdot (1.05 \cdot d - \frac{X_e}{3})} = \frac{1343000}{\frac{1}{2} \cdot 1000 \cdot 12.05 \cdot (1.05 \cdot 57 - \frac{12.05}{3})}$$

$$E_c \cdot \varepsilon_c = \frac{1343000}{336395.83} \rightarrow E_c \cdot \varepsilon_c = 3.99 \text{ N/mm}^2$$

Stress limitation

$$\sigma_{c,qp} = E_c \cdot \varepsilon_c \leq 0.45 \cdot f_{ck} \rightarrow \sigma_{c,qp} = 3.99 \leq 0.45 \cdot 25 = 11.25 \text{ is OK}$$

To prevent yielding of the steel at service load, Eurocode 2 specifies:

- Under rare load combination

$$\sigma_s = E_s \cdot \varepsilon_c \cdot \frac{d - X_e}{X_e} \leq 0.8 \cdot f_{yk}$$

$$\varepsilon_c = \frac{\sigma_{c,r}}{E_c} = \frac{7.97}{31000} = 2.57 \cdot 10^{-4}$$

$$\sigma_s = 205000 \cdot (2.57 \cdot 10^{-4}) \cdot \frac{57 - 12.05}{12.05} = 196.6$$

$$\sigma_s = 196.6 \leq 0.8 \cdot f_{yk} = 0.8 \cdot 500 = 400 \text{ is OK}$$

- Under quasi-permanent load combination

$$\sigma_f = E_f \cdot \left( \varepsilon_c \cdot \frac{h - X_e}{X_e} - \varepsilon_o \right) \leq \eta \cdot f_{fk}$$

$$\eta = 0.8 \text{ (for CFRP)}$$

$$\varepsilon_c = \frac{\sigma_{c, qp}}{E_c} = \frac{3.99}{31000} = 1.28 \cdot 10^{-4}$$

$$\sigma_f = 170000 \cdot \left[ (1.28 \cdot 10^{-4}) \cdot \frac{57 - 12.05}{12.05} - 6.84 \cdot 10^{-4} \right] \approx 0 \text{ N/mm}^2$$

$$\sigma_f = 0 \leq 0.8 \cdot f_{fk} = 0.8 \cdot 3100 = 2480 \text{ N/mm}^2 \text{ is OK}$$

As can be observed in the result, few deformations are expected in the reinforcement under service load, the creep rupture thereof is not a concern.

### Conclusions

	Q	G	q		Md	
Origin loads (kN/m <sup>2</sup> )	2,94	5,2	11,769	Origin moment	3,18	As,eff=1,57cm <sup>2</sup>
New loads (kN/m <sup>2</sup> )	2,94	6,76	14,109	New moments	3,81	As,nec=1,59cm <sup>2</sup>

With the original reinforcement, the slab didn't can support the new requirements ( $A_{s,nec} = 1.59 \text{ cm}^2 > 1.57 \text{ cm}^2 = A_{s,eff}$ ), so it was needed the strengthening with CFRP.

ULS checking		SLS checking	
$M_{rd} = 12.62 \text{ kNm} > M'd = 3.81 \text{ kNm}$	✓	$\sigma_{c, r} = 7.97 \leq 0.6 \cdot 25(f_{ck}) = 15$	✓
$f_{yd}/E_s = 2.12 \cdot 10^{-3} \leq 7.6 \cdot 10^{-3} = \varepsilon_s$	✓	$\sigma_{c, qp} = 3.99 \leq 0.45 \cdot 25(f_{ck}) = 11.25$	✓
$\varepsilon_f = 1.14 \cdot 10^{-2} \leq 1.8 \cdot 10^{-2} = \varepsilon_{fud}$	✓	$\sigma_s = 196.6 \leq 0.8 \cdot f_{yk} = 0.8 \cdot 500 = 400$	✓
$\xi = x/d = 0.31 < 0.45 = \xi(c25/30)$	✓	$\sigma_f = 0 \leq 0.8 \cdot f_{fk} = 0.8 \cdot 3100 = 2480$	✓

In the ULS checking, the equations of the yield strength of the steel working under tension ( $E_s/f_{yd}$ ) and the deformation of the CFRP laminate ( $\varepsilon_f$ ), less than the ultimate deformation ( $\varepsilon_{s1}$  and  $\varepsilon_{fud}$  respectively).  $\xi$  is to check the correct ductility of C25/30 concrete. All checking equations are correct.

In the SLS checking is necessary to prevent yielding of the steel at service under rare load combination ( $\sigma_s$ ) and the same with the CFRP under quasi permanent load combination. The others equations ( $\sigma_{c, r}$  and  $\sigma_{c, qp}$ ) are correct too, so won't be excessive compression, and won't appear longitudinal cracks and irreversible strains.

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