

Cognitive Multiple Criteria Decision Making and the Legacy of the Analytic Hierarchy Process^{*}

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ABSTRACT

This paper examines the concept, historical evolution and the different schools of thought regarding an aspect of one of the most productive and widely disseminated Operations Research disciplines of our time: Multiple Criteria Decision Making (MCDM). It analyses the development of the scientific method and new tendencies that have emerged in the context of the Knowledge Society. The work also considers future challenges in the field of MCDM, particularly in the context of one of the most popular approaches: the Analytic Hierarchy Process (AHP). Three fundamental problems envisioned for this school of thought by its creator need to be addressed as future challenges in the MCDM field. This article is dedicated to the memory of Professor Thomas L. Saaty, the originator of AHP and one of the most brilliant and ingenious mathematicians of the last 60 years; he passed away in August 2017 at the age of 91.

Keywords: Multiple Criteria Decision Making, Paradigms of Rationality, Cognitive Decision Making, Analytic Hierarchy Process, Group Decision Making, Conflict Resolution, Neural Activity.

Decisión Multicriterio Cognitiva y el Legado del Proceso Analítico Jerárquico

RESUMEN

Este trabajo recoge brevemente el concepto, la evolución histórica y las diferentes escuelas de una de las partes de la Investigación Operativa más fructífera y con mayor difusión de los últimos 45 años: la Decisión Multicriterio. Así mismo, analiza la evolución que el método científico ha seguido en este periodo de tiempo y cuáles son las nuevas orientaciones que presenta en el contexto de la Sociedad del Conocimiento. Finalmente, se incluyen una serie de ideas sobre cuáles pueden ser algunos de los retos futuros en el campo de la toma de decisiones multicriterio (TDMC), en particular en el contexto de una de las aproximaciones más populares: el Proceso Analítico Jerárquico (AHP). Tres problemas fundamentales, ya vislumbrados para esta escuela de pensamiento por su creador, necesitan ser abordados como futuros retos en el campo multicriterio. Este trabajo se ha dedicado a la memoria del profesor Thomas L. Saaty, el autor de AHP y uno de los matemáticos más brillantes e ingeniosos de los últimos 60 años, quien acaba de fallecer en agosto de 2017 a los 91 años de edad.

Palabras clave: Decisión Multicriterio, Paradigmas de Racionalidad, Decisión Cognitiva, Proceso Analítico Jerárquico, Decisión en Grupo, Resolución de Conflictos, Actividad Neuronal.

Clasificación JEL: C02, C18, C44, C54, C65, D72, D83

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“In memory of our beloved mentor and friend Professor Thomas L. Saaty”

1. INTRODUCTION

The ability to *make decisions* is an inherent and essential characteristic of human beings that reflects their degree of evolution, knowledge and freedom. Until the mid-1970s, scientific decision making was based on *the paradigm of substantive rationality*, a mechanistic approach that only contemplated the objective part of reality (neutrality of values), ignoring the subjective, intangible and emotional factors associated with human beings (Söderbaum, 1999). Substantive rationality's orientation toward the result - the decision or 'product' - considered a single criterion (objective) and focused on the development of procedures and tools that facilitated finding the best decision or optimal solution.

Over the years, this myopic vision of reality has been corrected in order to adjust it to the behaviour of individuals and the systems in which they are immersed. In the past, a lack of technical knowledge to consider multiple criteria and subjective factors meant that the scientific method exclusively concentrated on a single-criterion, objective analysis of reality. In today's society, there is no excuse for continuing to make this mistake with the technology and tools now available (Moreno-Jiménez, 2016).

The scientific treatment of decision making problems involving both tangible and intangible factors, multiple scenarios, actors and criteria is known as *Multiple Criteria Decision Making* (MCDM). It originated in the early 1970s and has been one of the most active and productive areas in Operations Research. Furthermore, due to its transversal character, it has also been one of the most commonly applied in all fields of scientific knowledge.

One of the most interesting contemporary challenges facing the science of decision making (**Challenge #1**) is the integration of *reason* and *emotion* into the processes associated with the traditional scientific methodology. The two aforementioned concepts are fundamental characteristics of the human factor, a key element in the Knowledge Society (Moreno-Jiménez *et al.*, 1999).

This paper is based on a response to this challenge and others that are put forward in Section 4. The work is structured as follows: Section 2 discusses what is meant by MCDM; Section 3 briefly outlines the evolution of the scientific method; Section 4 considers the scientific methodology required for dealing with the challenges posed by the 21st century and the Knowledge Society (KS); Section 5 includes three fundamental problems in the Analytic Hierarchy Process (AHP) school that need to be addressed as challenges in the MCDM field, and, Section 6 presents some conclusions regarding the most significant issues that will have to be faced.

2. MULTIPLE CRITERIA DECISION MAKING

From its inception, almost 50 years ago¹, MCDM has been based on the resolution of complex decisional problems involving multiple scenarios, actors and criteria (tangible and intangible). Its aim is to deal with these problems in a more realistic and effective manner than it is possible with single-criterion models of traditional science that only incorporate the objective part of reality.

MCDM is focused on providing an appropriate response to Challenge #1: the development of a *new scientific methodology* that can integrate the tangible, objective and rational with the intangible, subjective and emotional. Despite the complexity of its implementation, the solution is relatively simple; the objective treatment of the subjective (Keeney, 1992) or, to put it another way, a rational treatment of the emotional (Moreno-Jiménez *et al.*, 2012).

In the past, MCDM could be understood as: (1) the possibility of establishing a balanced analysis of problems of planning, especially those that involved intangible aspects such as social and environmental concerns (Nijkamp and van Delft, 1977); (2) the consideration of a number of alternatives in the context of multiple criteria and conflicting objectives (Voogd, 1983); (3) a set of models, methods and techniques that would enable the decision makers to describe, evaluate, order, rank and select or reject objects based on an evaluation (Colson and de Bruin, 1989); and, (4) the resolution of complex decision making problems which could involve multiple criteria and objectives (Romero, 1993). Currently, the objective of MCDM is considered as being the facilitation of the process of scientific decision making (Saaty, 1994).

In highly complex problems, what is unknown is much greater than what is known. In these cases, it is preferable to replace traditional science's *search for the truth* (unique, universal and objective) with *the search for knowledge* (tendencies, patterns and critical points) and its *diffusion* (arguments that justify opinions and decisions), a process that is associated with the new scientific methodology (Moreno-Jiménez, 2006; Moreno-Jiménez *et al.*, 2012, 2014).

In line with this interpretation of scientific methodology, MCDM can be defined as the set of approximations, methods, models, techniques and tools that aim to: (i) increase our knowledge of decision making processes - knowledge about the Products, Processes and People (3Ps) involved in the decision (Moreno-Jiménez *et al.*, 2012); and, (ii) improve the scientific quality of the resolution processes, i.e., their *effectiveness* (the degree to which something is successful in producing a desired result) and *efficiency* (the ability to accomplish goals with the least waste of resources).

¹ It is generally agreed that the discipline was formally recognised in 1972, the year in which the first Conference on Multiple Criteria Decision Making took place in South Carolina (USA).

3. PARADIGMS OF RATIONALITY

Paradigms of rationality represent a scientific approach to decision making. The three most commonly employed paradigms are (Moreno Jiménez *et al.*, 1999): *substantive rationality* (rational decision maker), *bounded rationality* (satisficing decision maker) and *procedural rationality* (descriptive decision maker).

Substantive rationality dominated the field of decision making from the time of its appearance in the middle of the 20th century (Savage, 1954). It is a strict approach characterised by its optimization behaviour (maximum well-being). It is product-oriented (output or decision) and based on having knowledge of the alternatives, their consequences and the criteria followed by the evaluation and comparison of those alternatives. It is a normative approach guided towards prediction and control that explains how the decisions should be made.

Bounded rationality emerged at the end of the 1960s (Simon, 1972) as a response to the cognitive limitations of human beings, e.g., ignorance, stupidity and passion (Kaufman, 1999). It is based on two concepts: 'search' and 'satisficing'. The former is associated with the lack of knowledge of the alternatives whiles the latter refers to achieving goals set by the objectives.

Procedural rationality materialised in the 1970's via the behavioral and sociological approaches to economic decision making (Kanheman and Tversky, 1979). It is oriented towards the process, and it is practical, realistic and formative. Its aim is twofold: understanding and consensus (Moreno Jiménez *et al.*, 1999). Decision making models based on procedural rationality consider intangible and subjective aspects that condition the decisions of individuals and organizations.

The two most common schools of thought regarding decision making are: (i) *normative* (based on substantive rationality), a strict approximation oriented to the *product* that indicates how decisions should be taken and the methods that should be used; and (ii) *descriptive* (based on procedural rationality), a 'soft' approximation oriented to the *process* that indicates how decisions are taken. At the end of the 1980s (Tversky, 1988), the *prescriptive* school advanced a 'constructivist' approximation, oriented towards the knowledge that indicates how to improve decision making processes.

The prescriptive school utilises new paradigms of rationality such as Soft Systems, Post-normal Science, Postmodernism, Critical Realism, and Multicriteria Procedural Rationality, among others. Due to its suitability for the adaptive and evolutionary focus suggested in Section 4 regarding Multiple Criteria Decision Making, the remainder of this article concentrates on the *Paradigm of Multiple Criteria Procedural Rationality* (MCPR) (Moreno-Jiménez *et al.*, 1999).

In conformance with the three factors considered in the specification of the paradigm (Tacconi, 2000), MPR is determined (Moreno Jiménez *et al.*, 2001) by its *relativist and emotional ontology*, its *adaptive epistemology* and its *cognitive constructivist methodology*.

Therefore, the new approach is descriptive, cognitive, adaptive, pragmatic and systemic. It aims to facilitate scientific decision making by offering improved scientific rigour in each of the stages and phases followed in the resolution process, thereby increasing knowledge of the process itself. This is achieved by having greater knowledge of the elements involved: scenarios, criteria, factors, interdependencies, actors, interrelationships, techniques and procedures.

MCPR is an approach oriented towards the person (P), the analysis focuses on: (1) understanding the decisional process; (2) increasing the value added by the knowledge gained in the resolution of the problem, particularly with regards to justification and learning; (3) identifying the critical points, patterns and decisional opportunities that generate the formulation of new alternatives; (4) discovering the preferences of the actors - this is of special importance in the feedback phase; and, (5) strengthening the processes of negotiation and dialogue.

MCPR consists of the following steps or phases:

- (i) Problem Formulation and Description
- (ii) Model Development
- (iii) Preference Elicitation
- (iv) Prioritization and Synthesis
- (v) Uncertainty, Robustness and Feedback
- (vi) Implementation of the Solution, Negotiation and Learning.

Phases (ii)-(iv) correspond to essential stages of AHP, the remaining phases reflect the cognitive orientation of the MCPR.

The use of AHP as the methodological support of MCPR is because of its: (1) intuitive and realistic character in scientific decision making; (2) ability to integrate through hierarchies and clustering the large and the small; (3) capability of combining tangible and intangible aspects of problems by means of absolute pairwise comparisons that yield relative ratio scales of priorities; (4) flexibility to consider dependencies between levels in a hierarchy with the extension of the AHP known as ANP (Analytic Network Process); (5) power in group decision making allowing decision makers the construction of group welfare functions that do not violate Arrow's conditions; and (7) strength in negotiations and learning / cognition (discussion, extraction and dissemination of knowledge).

4. COGNITIVE MULTIPLE CRITERIA DECISION MAKING

Science, in a traditional sense, is the knowledge of universal validity characterised by objectivity (the neutrality of values), causality (the construction of an explicative model), rationality (the use of logical and coherent procedures) and verifiability (the results can be reproduced and tested). Science searches for logical relations and causal connections in structures of homogeneous entities, based on two philosophical hypotheses associated with the objective description of reality: (i) the existence of a reality to describe (ontological presumption), and (ii) the possibility of achieving a universal knowledge of that reality, independent of personal values, emotions and views (epistemological presumption).

At any given point in history, the scientific method followed is dependent on the characteristics of society at that time. At present, the context that conditions our activities is the so-called *Knowledge Society* (KS), which can be understood as a space for the talent, imagination and creativity of human beings. Aided by the development of Information and Communication Technologies (ICTs), the *Knowledge Society* has three defining characteristics: (i) the interdependency between the factors and the interrelationships between actors (*a holistic vision* of reality); (ii) improved education and training and the collaborative readiness of the actors (*aptitude* and *attitude*); and, fundamentally, (iii) the importance of the *human factor*: the explicit consideration of the intangible, subjective and emotional.

Given that a key element of the KS is the human factor and its holistic perspective, the new scientific method must understand and reflect this idea. The point of reference is, therefore, the evolution of living systems characterised by three elements (Capra, 1996): Pattern, Structure and Process. *Pattern* being Maturana and Varela's autopoiesis (self-organisation); *Structure* refers to the dissipative structures (order in the disorder) of Ilya Prigogine; and *Process* corresponds to the vital process of living beings - the cognitive process.

This cognitive process is founded on the *plurality of opinions*, the *diversity of ideas* and the *personal selection* between them. The process is able to foster the subsistence and evolution of humankind in a manner which is analogous to the process of genetic diversity and the natural selection of living systems that has functioned for thousands of years. Only species that learn and adapt to their context survive.

In this framework, *MCDM must have a cognitive orientation (Challenge #2)*. It must be aimed at the continuous education of individuals (and the systems in which they are immersed) in that distinctive aspect of human beings - the ability to make decisions ('scientifically', in this case). The new methodology must add a further stage to the stages that are traditionally included in the scientific resolution of problems: *cognition*, both individual and societal. It is not enough to reach the optimum decision or solution (the *product*), or increase the knowledge and

rigour of the resolution *process*; there must be an orientation towards improving the knowledge of *people*.

The response to this challenge (#2) requires the systematization of the cognitive exploitation of decisional processes (Moreno-Jiménez *et al.*, 2014). There are four main stages: i) the detailed *Formulation* of the problem, specifying all the relevant elements from a cognitive point of view; ii) the *Discussion* by the actors involved in the resolution of the problem (a discussion stage between two voting rounds in which the preferences of decision makers are incorporated); iii) the *Exploitation* of the mathematical model to extract the maximum knowledge possible (patterns, critical points, decision opportunities, arguments that support positions etc.); and, iv) the *Diffusion* (including *Visualisation*) of knowledge. In addition, as a starting point for learning and continuing education, a fifth stage, *Accountability*, is advisable. This is an evaluation of the efficiency (doing things correctly), the efficacy (achieving goals) and, in particular, the effectiveness (doing what is right in order to resolve the problem) of the resolution process.

Challenge #3 is the implementation of the *Discussion* stage contemplated in the cognitive orientation. As an example, for this new stage, a systematic procedure must be established that allows us to: (i) take advantage of the talent and experience of actors; (ii) link the arguments with the preferences; (iii) incorporate quantitative information and qualitative knowledge; (iv) measure the *individual importance* and *social relevance* of the themes (messages and comments) as well as the *individual confidence* and *social reputation* of the participating actors; (v) evaluate the degree of compatibility between the individual and collective positions; (vi) determine the discrepancy thresholds which can be the basis for a new order in situations that are distant from the equilibrium (*social dissipative structures*); (vii) incorporate social networks into the electronic participation (e-participation) processes, and, (viii) guarantee the levels of security demanded by e-discussion and e-decision procedures.

The *Exploitation* of the mathematical model (in our case, AHP) and the information and knowledge generated in the *Discussion* stage allow us to: (a) measure the changes in collective and individual preferences; (b) extract the arguments that support the opinions and decisions; (c) identify the social leaders and most significant themes; and, most importantly, (d) measure the value added by the increase in individual and collective knowledge produced by the technique that is employed. This measurement can be used to determine *the most suitable multiple criteria approach* for each case (**Challenge #4**): the approach that provides the greatest added value to the system. It should be emphasised that the continuous education of people and the systems in which they are involved is the foundation of this new cognitive approach.

The KS is characterised by the development of ICTs, the holistic vision of reality and the importance of human factor. In this context, the scientific methodology for the resolution of complex problems must encompass the intangible, subjective and emotional elements that are associated with people and the cognitive orientation that characterises the vital processes of living beings.

The incorporation of emotional elements (Challenge #1) and a cognitive orientation (Challenge #2) into decision making processes have been the principal objectives contemplated in this work. One of the keys to determining an appropriate response to such challenges is increasing our understanding of the *functioning of the human brain* when it has to process existing information and knowledge in order to make decisions. This new challenge (**Challenge #5**) is the study object of innumerable research projects (Glimcher and Fehr, 2014) in a wide range of fields (neuroscience, psychology, decision making, economics, computation, communication, etc.).

From the perspective of decision sciences, this challenge involves the mathematical modelling of the nervous system, in terms of both decision making and processing information. Other examples of similar attempts are Gómez and Ríos-Insua (2017) studying the construction of a robot that incorporates an affective “utilitarian” model with a variety of emotions, and two Canadian works on the Bayesian modeling of emotions (Obeidi *et al.*, 2005; García, 2014).

5. THE LEGACY OF THE ANALYTIC HIERARCHY PROCESS

One of the most difficult tasks that modelers face is the incorporation of human behavior into decision making. It is known that human behavior is not always rational in the way it is assumed by the rational choice school. In recent years a new way of thinking has evolved using psychology and economics that is trying to show that transitivity need not always be satisfied to be a rational decision maker.

Kahneman and Tversky (1979) showed the many problems that expected utility theory has as a descriptive theory of behavior leading to preference reversals; and Tversky and Thaler (1990) provided some plausible explanations as to how preference reversals may occur when people make decisions. Richard Thaler, the 2017 Nobel Prize in Economics, has demonstrated that mankind is afflicted by emotion and irrationality, which influences their decision making on everything from retirement savings, to health-care policy, to professional sports. This is in complete agreement with what Thomas L. Saaty has been saying for years.

Saaty’s theory (1977, 1980, 1986) the Analytic Hierarchy Process (AHP), is based on the idea that making decisions need not assume transitivity. One could

go one step further and imply that the lack of transitivity in preferences may lead to rank reversals. It is one of the reasons why Saaty's theory has been criticized. However, a theory of decision making should allow for intransitivity if we expect to capture what Thaler (2017) calls "predictably irrational" behavior. Thaler does not believe that human beings are randomly irrational. He is not the only one who believes this to be the case. Ariely (2008) also challenges the assumptions about making decisions based on rational thought.

We believe we make decisions by comparing alternatives in pairs according to different criteria, but for every pairwise comparison, we only have one and only one criterion in mind. We perform all the comparisons according to all the criteria, and somehow we synthesize all the comparisons in our brain to arrive at the final decision. Saaty created his theory to help model this process and incorporate the experience, talent and knowledge of the actors involved in the resolution process. However, as a theory of decision making, the AHP, and its extension to networks the Analytic Network Process (ANP), approximates how we actually make decisions.

AHP provides the flexibility of accepting or rejecting transitivity in the modeling process. We know that a necessary and sufficient condition for rank preservation in the AHP is row dominance (Saaty and Vargas, 2012). Nonetheless, people are not always transitive, and hence, they violate a fundamental premise of the rational choice school and, Thaler's "predictably irrational" behavior follows.

When Saaty conceived AHP, he envisioned three fundamental problems that needed to be addressed (**Challenge #6**):

(1) Group decision making.

Now more than ever, group decision making is critical at all societal levels. Problems are becoming more complex requiring multiple experts to understand all dimensions of problems, and the implications of decisions are multidimensional. We need to be able to make decisions in groups without the fear of having a decision being imposed on us. This would be the case, if all we do is ranking alternatives, because then we could fall under the umbrella of Arrow's Impossibility theorem. We need to ensure that a decision by a group is not a dictatorial one. Saaty and Vargas (2012) showed that it is possible to make decisions in groups without being dictatorial if intensity of preference given by the individual judgments is represented with an absolute scale, and the social welfare function is a ratio scale derived from the geometric mean of the individual judgments.

(2) Conflict resolution

In 1981 the book "Getting to YES" (Fisher and Ury, 1981) revolutionized the way conflicts were looked at. Fisher and Ury introduced the concept of

principled negotiation in which participants are problem solvers. The approach is based on four principles: (i) Separate the people from the problem; (ii) Focus on interests not positions; (iii) Invent options for mutual gain, and (iv) Insist on using objective criteria. In this approach, parties do not see each other as adversaries but rather as collaborators in search of a fair solution.

However, the approach does not measure gains and losses of parties for different options. Thus, the parties may not be able to perceive how fair a proposed solution is. What is needed is the development of scales that represent the preferences of the parties. It is not enough to assign numbers to preferences without any mathematical assumptions because we want to ensure that the results belong to a measurement scale.

This is a difficult problem if the dimensions of the conflict involve intangibles, which by definition are considered not to have a scale of measurement. Pairwise comparisons from Saaty's *absolute scale* (Saaty, 1977) can be used to build such relative measurement scales. In Saaty *et al.* (2017), this approach was used to show that a fair solution (in the eyes of those involved in the process) could be developed.

This is just one of many examples that show that to deal with conflicts, the negotiation approach needs to be measurement based. Since intangibles are always involved, we need pairwise comparisons to build measurement scales, that are then used to compute gain/loss ratios of tradeoffs from each party's perspective. Gain/loss ratios are not symmetric and the tradeoffs are non-zero sum. Hence, measurement allows for the selection of tradeoffs for which both parties benefit equally through a MaxMin optimization model.

(3) Pairwise comparisons and neural activity

The nervous system uses its own kind of mathematical function patterns to deal with both external and internal realities. The conscious part of the nervous system is there to respond to what happens outside by regulating externally received information signals from the senses and the skin and muscles of the body itself. To do that, it needs to communicate with its subconscious using the familiar language of neural firing. Saaty and Vargas (2017) show that because reciprocal pairwise comparisons are performed at the neural level, the division algebra of octonions (Baez, 2001), in which commutativity and associativity are not satisfied, provides the structure needed to represent mental processes.

Saaty showed, while extending the discrete pairwise comparisons to continuous spaces, that the response of a neuron in spontaneous activity, $w(s)$, is an eigenfunction solution of a Fredholm's integral equation of the second kind if and only if it satisfies the functional equation $\mathbf{w}(as) = b\mathbf{w}(s)$, where s represents stimuli (Saaty, 2015; 2017a,b). Saaty called this equation the fundamental equation of pairwise comparisons. Its solution in the space of

octonions is given by $\tilde{w}(u) = a^{\left(\frac{\ln b}{\ln a}\right)u} P(u) \oplus P(u) a^{\left(\frac{\ln b}{\ln a}\right)u}$, where $P(u)$ is a periodic function of period 1. It satisfies the condition $\tilde{w}(uv) = \tilde{w}(u)\tilde{w}(v)$ if $P(u)$ satisfies the semigroup condition $P(u+v) = P(u)P(v)$, and it can generate the group of automorphisms, G_2 .

In G_2 , these functions are given by $\tilde{w}(u) = b^u e^{2\pi i u}$, and they are dense in the space of continuous functions defined on the octonions. Thus, all continuous functions could be expressed as linear combinations of the solution of the equation, and they could generate the group of automorphisms. In sum, any representation of brain activity with octonions could be expressed with the solution of the equation $\mathbf{w}(as) = b\mathbf{w}(s)$. According to this result, the firing of neurons through the continuous paired comparison process generates a smooth G_2 -manifold in which cognition and the representations of our thoughts could take place (Saaty and Vargas, 2017).

6. CONCLUSIONS

In this paper, we have put forward a program related to emotion and cognition for what we think MCDM in general, and AHP in particular, needs to consider incorporating human behavior in decision making models. This program consists of six challenges:

1. The integration of *reason* and *emotion* into the processes associated with the scientific resolution of decisional problems;
2. The need for MCDM to have a *cognitive orientation*;
3. The *implementation of the new stage* (discussion stage) contemplated in the cognitive orientation;
4. The determination of *the most suitable multicriteria approach* for each case based on knowledge added value;
5. The need to increase our understanding of the *functioning of the human brain* when it must process existing information and knowledge to make decisions; and
6. Three concrete problems in which cognitive processes play a fundamental role: *group decision making, conflict resolution and negotiation* and the *role of pairwise comparisons in brain activity* in the process of decision making.

From a methodological point of view, two of the most relevant contributions of AHP to decision making has been: the development of ratio scales from absolute comparisons, used to measure “intangibles” in relative terms, and the relaxation of the axiom of transitivity to capture “predictably irrational” behavior.

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